Homework 6 - Math 300 C - Spring 2016 - Dr. Matthew Conroy

1. Let A, B and C be sets. Let $f : A \to B$ and $g : B \to C$.

   (a) Prove that if f and g are onto, then $g \circ f$ is onto.

   (b) Prove that if $g \circ f$ is onto, then g is onto.

   (c) If $g \circ f$ is onto, is f necessarily onto? Prove your answer.

2. Suppose A, B and C are sets. Suppose $f : A \to B$ and $g : B \to C$.

   (a) Prove that if f is onto and g is not one-to-one, then $g \circ f$ is not one-to-one.

   (b) Prove that if f is not onto and g is one-to-one, then $g \circ f$ is not onto.

3. Let $f : A \to B$ and $g : B \to A$. Suppose $g \circ f = i_A$. Then f is one-to-one and g is onto.

4. Let a and b be integers. Let $f : \mathbb{Z} \to \mathbb{Z}$ be defined by $f(x) = ax + b$. Determine the conditions on a and b necessary and sufficient for f to be a bijection, and then use those conditions to complete, and then prove, the following statement: "f is a bijection iff ______________".

5. Let A and B be sets, and $f : A \to B$. Suppose f is one-to-one. Prove that there exists a subset $C \subseteq B$ such that $f^{-1} : C \to A$.

6. Prove the following theorems about rational and irrational numbers.

   (a) The sum of an irrational number and a rational number is an irrational number.

   (b) The product of an irrational number and a non-zero rational number is an irrational number.

   (c) The sum of two irrational numbers may be rational (unlike the two theorems above, here an example is sufficient since the theorem is equivalent to the statement that there exists at least two irrational numbers whose sum is rational)

7. Prove that $\sqrt{2} + \sqrt{7}$ is irrational (hint: suppose it is rational, and show that this implies that $\sqrt{2}$ is rational).