Homework 1 - Math 300 C Spring 2016 - Dr. Matthew Conroy

1. Prove each of the following theorems from the axioms of the integers. Use a Theorem/Proof format for each one.

   (a) Let a and b be integers. Then $(a + b)^2 = a^2 + 2ab + b^2$.

   (b) Let a be an integer. Then $a(-1) = -a$. (This is EPI #3.)

   (c) (-1)(-1)=1. (You might want to first prove that $-(-a) = a$ for any integer a, but it is not needed.)

   (d) Let a and b be integers. Then $(-a)(-b) = ab$. (This is EPI #5.)

   (e) Let a, b and c be integers. Suppose $a < b$. Then $a + c < b + c$. (This is EPI #9.)

2. Define the *absolute value* of an integer x to be

$$|x| = \begin{cases} x & \text{if } x \geq 0, \\ -x & \text{if } x < 0. \end{cases}$$

   Write proofs of the following two theorems.

   (a) Let $x \in \mathbb{Z}$. Then $|x| = |-x|$.

   (b) Let $x \in \mathbb{Z}$. Then $x^2 = |x|^2$.

3. Prove the following two theorems.

   (a) The sum of two odd integers is even.

   (b) The product of two odd integers is odd.