MATH 300 B Winter 2015
Midterm 1 Study Problems

1. Give a useful denial (i.e., negation) of each statement. You may use symbols like $\forall$ and $\exists$, et cetera.

   (a) For every real number $x$, there exists a real number $y$ such that $x < y$.
   (b) There exists a function $f : \mathbb{R} \to \mathbb{R}$ such that, for every real number $x$, $f(2x) = 4f(x)$.
   (c) If $x$ is a positive real number, then $\ln x \geq 0$ or $\frac{1}{x} > 1$.
   (d) For all real $x$, $x \neq 0 \Rightarrow x^2 > 0$.

2. Let $P$ be the statement: "Every prime number is odd." Which of the following are logically equivalent to $P$? (Check all that apply.)

______ $n$ is odd implies $n$ is prime.
______ $n$ is odd if $n$ is prime.
______ If $n$ is odd, then $n$ is prime.
______ If $n$ is prime, then $n$ is odd.
______ There exist numbers that are both prime and odd.
______ No even number is prime.
______ $n$ is odd if and only if $n$ is prime.

3. Show the following set equalities.

   (a) $(A \cup B) \cup (A \cap B) = A \cup B$
   (b) $(A \setminus B) \cup (B \setminus A) \cup (A \cap B) = A \cup B$
   (c) $(A \setminus B) \cap (A \cap B) = \varnothing$
   (d) $(A \cap B) \cap A = (A \cap B)$

4. Show the equivalence of the following statement pairs.

   (a) $(P \vee Q) \Rightarrow R$ and $\neg R \Rightarrow (\neg P \wedge \neg Q)$
   (b) $(P \Rightarrow Q) \Rightarrow R$ and $(P \wedge \neg Q) \vee R$
   (c) $P \Leftrightarrow Q$ and $(\neg P \vee Q) \wedge (\neg Q \vee P)$

5. Use quantifiers to write each of the following statements.

   (a) Every integer can be written as the sum of two squares.
   (b) There exists a real number $r$ such that $rx = 0$ for every real number x.
   (c) For every real number $x$, we can find exactly one real number $y$ such that $x + y = 0$.

6. Write the negation of each of the following. Is each statement true or false? For all statements, the universe is $\mathbb{Z}$ (that is, assume all variables represent integers).

   (a) $\exists x \exists y((xy = 100) \wedge (x + y = 30))$
   (b) $\forall x \exists! y(x + y = 7)$
   (c) $\exists a \forall b(ab = b)$