Homework 7 - Math 300 B - Winter 2015 - Dr. Matthew Conroy
Relevant reading: Velleman, 5.1, 5.2, 5.3, 7.1, 7.2

1. Define a function $f : \mathbb{R} \to \mathbb{R}$ by

$$f(x) = \begin{cases} 2x & \text{if } x \in \mathbb{Q} \\ -3x & \text{if } x \notin \mathbb{Q} \end{cases}$$

   Is f one-to-one? Is f onto? Is f^{-1} a function? State and prove a theorem.

   (a) Show that f is one-to-one and onto.

   (b) Give a formula for $f^{-1}(x)$.

2. Let A, B and C be sets. Let $f : A \to B$ and $g : B \to C$.

   (a) Prove that if f and g are onto, then $g \circ f$ is onto.

   (b) Prove that if $g \circ f$ is onto, then g is onto.

   (c) If $g \circ f$ is onto, is f necessarily onto? Prove your answer.

3. Let A be the set of subsets of $\mathbb{R}$. Define a function $f : \mathbb{R} \to A$ by

$$f(x) = \{z \in \mathbb{R} : |z| > x\}.$$

   Is f one-to-one? Is f onto?

4. Let A and B be sets, and $f : A \to B$. Suppose f is one-to-one. Prove that there exists a subset $C \subseteq B$ such that $f^{-1} : C \to A$.

5. For each of the following pairs of sets, give a bijection from the first set to the second set. Then give the inverse of each bijection.

   (a) $\mathbb{Z}$ and $\mathbb{Z} \setminus \{-6, 0, 5\}$

   (b) $(-2, \infty)$ and $(-\infty, 7)$ (these are intervals, i.e., subset of $\mathbb{R}$)

   (c) $(-\infty, 3)$ and $(0, 1)$ (these are intervals, i.e., subset of $\mathbb{R}$)

6. Let A and B be finite sets. If $A \cap B = \varnothing$, then $|A \cup B| = |A| + |B|$.

7. Let A be a finite set. Prove that if $f : A \to A$ is injective, then f is bijective.

8. Prove that, if $A \sim B$, then $\mathcal{P}(A) \sim \mathcal{P}(B)$.