1. Define a function \( f : \mathbb{R} \to \mathbb{R} \) by
\[
f(x) = \begin{cases} 
2x & \text{if } x \in \mathbb{Q} \\
-3x & \text{if } x \notin \mathbb{Q}
\end{cases}
\]
Is \( f \) one-to-one? Is \( f \) onto? Is \( f^{-1} \) a function? State and prove a theorem.

(a) Show that \( f \) is one-to-one and onto.
(b) Give a formula for \( f^{-1}(x) \).

2. Let \( A, B \) and \( C \) be sets. Let \( f : A \to B \) and \( g : B \to C \).

(a) Prove that if \( f \) and \( g \) are onto, then \( g \circ f \) is onto.
(b) Prove that if \( g \circ f \) is onto, then \( g \) is onto.
(c) If \( g \circ f \) is onto, is \( f \) necessarily onto? Prove your answer.

3. Let \( A \) be the set of subsets of \( \mathbb{R} \). Define a function \( f : \mathbb{R} \to A \) by
\[
f(x) = \{ z \in \mathbb{R} : |z| > x \}.
\]
Is \( f \) one-to-one? Is \( f \) onto?

4. Let \( A \) and \( B \) be sets, and \( f : A \to B \). Suppose \( f \) is one-to-one. Prove that there exists a subset \( C \subseteq B \) such that \( f^{-1} : C \to A \).

5. For each of the following pairs of sets, give a bijection from the first set to the second set. Then give the inverse of each bijection.

(a) \( \mathbb{Z} \) and \( \mathbb{Z} \setminus \{-6, 0, 5\} \)
(b) \((-2, \infty)\) and \((-\infty, 7)\) (these are intervals, i.e., subset of \( \mathbb{R} \))
(c) \((-\infty, 3)\) and \((0, 1)\) (these are intervals, i.e., subset of \( \mathbb{R} \))

6. Let \( A \) and \( B \) be finite sets. If \( A \cap B = \emptyset \), then \(|A \cup B| = |A| + |B|\).

7. Let \( A \) be a finite set. Prove that if \( f : A \to A \) is injective, then \( f \) is bijective.

8. Prove that, if \( A \sim B \), then \( \mathcal{P}(A) \sim \mathcal{P}(B) \).