Math 300 B - Winter 2015 - Homework 6
Problems on functions and induction
Relevant reading: Velleman, 5.1, 6.1

1. Find the smallest integer k such that $n! > 3n^4$ for all $n \geq k$. Use induction to prove this result, and be sure to also prove you have the smallest k.

2. Use induction to prove that $5^n > 10(4^n + 3^n)$ for all integers $n \geq 11$.

3. Let $a \in \mathbb{R}_{\neq 0}$. Use induction to prove that, for all integers $n \geq 0$,
$$\sum_{i=0}^{n} a^i = \frac{1 - a^{n+1}}{1 - a}.$$

4. Use induction to prove that
$$15 \mid 3^{4n} + 2^{12n+1} - 8$$
for all n in $\mathbb{Z}_{\geq 1}$.

5. Let n be a positive odd integer.

   Use induction to prove that the sum of all positive odd integers less than or equal to n is $\left(\frac{n+1}{2}\right)^2$.

6. Suppose $f : A \to C$ and $g : B \to C$. Prove that if $A \cap B = \varnothing$, then $f \cup g : (A \cup B) \to C$.

7. Suppose R is a relation on a set A. Is it possible that R is both a function and an equivalence relation? Answer this question by completing and proving the statement "R is a function and an equivalence relation iff ...".

8. Let S and T be sets and $f : S \to T$. Define a relation R on S by
$$(a, b) \in R \Leftrightarrow f(a) = f(b).$$
Prove that R is an equivalence relation.