Homework 4 - Math 300 B Winter 2015 - Dr. Matthew Conroy
Relevant readings: Velleman, sections 3.3, 3.4, 3.5, and 3.6.

1. Let a, b, c and d be integers, with $bd \neq 0$. Then $a\sqrt{b} + c\sqrt{d}$ is an algebraic number.
2. Let a and b be integers. Then $a^2b + a + b$ is even if and only if a and b are both even.
3. (a) Let n be an integer. Then the remainder when n^2 is divided by 4 is 0 or 1.
   (b) The numbers in the set $\{99, 999, 9999, ...\}$ cannot be written as the sum of two squared integers, but at least one can be expressed as the sum of three squared integers.
4. Let A and B be sets. Then $A \subseteq B$ iff $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.
5. Let A and B be sets. Then $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$, with equality if and only if $A \subseteq B$ or $B \subseteq A$. (You will need to show two things: (1) $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$, and (2) $\mathcal{P}(A) \cup \mathcal{P}(B) = \mathcal{P}(A \cup B)$ if and only if $A \subseteq B$ or $B \subseteq A$.)
6. Suppose $\mathcal{R}$ and $\mathcal{S}$ are families of sets. If $\mathcal{R} \subseteq \mathcal{S}$, then $\cup\mathcal{R} \subseteq \cup\mathcal{S}$.
7. Suppose $\mathcal{R}$ and $\mathcal{S}$ are families of sets, and $\mathcal{R} \neq \varnothing$ and $\mathcal{S} \neq \varnothing$. If $\mathcal{R} \subseteq \mathcal{S}$, then $\cap\mathcal{S} \subseteq \cap\mathcal{R}$.
8. Suppose $\mathcal{R}$ and $\mathcal{S}$ are families of sets. Then $(\cup\mathcal{R}) \setminus (\cup\mathcal{S}) \subseteq \cup(\mathcal{R} \setminus \mathcal{S})$.