Homework 3 - Math 300 B - Winter 2015 - Dr. Matthew Conroy
Relevant readings: Velleman, sections 3.1, and 3.2.

1. Let a and b be negative real numbers. Prove that if $a < b$ then $a^2 > b^2$.
2. Let a, b and c be positive integers. Prove that if $a|b$ and $b|c$, then $a|c$.
3. Let a, b, and c be integers, $c \neq 0$. If $ac|bc$, then $a|b$.
4. One fact we use all the time when writing proofs is that, if $A \to B$ and $B \to C$, then $A \to C$. Prove this is valid by showing that

$$((A \to B) \wedge (B \to C)) \to (A \to C)$$

is a tautology. Do this by using applicable laws, or a truth table, to show that this is a tautology (this is the last time we'll doing something like this this quarter, I promise!)

5. Now that we know the irrational numbers exist, we should prove a few facts about them.

You can use the following useful facts in your proofs. You do not have to prove them.

Fact 1: The sum of rational numbers x=a/b and y=c/d is (ad+bc)/(bd).

Fact 2: If a is rational, then -a is rational; if a is irrational, then -a is irrational.

Prove the following theorems:

(a) The sum of two rational numbers is a rational number.
(b) The sum of a rational number and an irrational number is an irrational number.
(c) The product of an irrational number and a non-zero rational number is an irrational number.
(d) The sum of two irrational numbers may be a rational number.