

Homework 1 - Math 300 D Autumn 2014 - Dr. Matthew Conroy  
Relevant readings: Velleman, sections 1.1, 1.2, 1.3.

1. Introduce variables and express the following sentences symbolically.

- (a) I will go to London or Paris, but not both.
- (b) Either Dave, Jing or Maria is lying, or they are all lying.
- (c) Peter and Xia are not both over two meters tall.
- (d) Peter and Xia are both not over two meters tall.
- (e) Olga is rich and famous, or she is not rich.
- (f) There is no cake left and I am hungry, or there is cake left but I am not hungry.

2. Write grammatical english sentences with the following structures.

- (a)  $(P \wedge Q) \vee \neg P$
- (b)  $\neg(D \wedge F) \vee F$
- (c)  $\neg(G \vee H) \wedge G$

3. Make truth tables of each of the following.

- (a)  $P \wedge \neg Q$
- (b)  $(P \wedge Q) \vee \neg P$
- (c)  $(P \vee Q) \wedge (\neg P \wedge Q)$
- (d)  $(P \vee Q) \wedge (P \vee R)$

4. Create truth tables for each of the following. Are any of them equivalent?

- (a)  $A \wedge \neg B$
- (b)  $\neg(A \vee \neg B)$
- (c)  $(A \wedge B) \vee (\neg A \wedge B)$
- (d)  $\neg A \vee (A \wedge \neg B)$
- (e)  $\neg(\neg A \vee (B \vee A))$

5. Simplify the following expressions.

- (a)  $(P \vee (\neg P \wedge P)) \wedge \neg P$
- (b)  $(P \wedge (Q \wedge R)) \vee (P \wedge R)$
- (c)  $\neg(P \wedge (\neg P \wedge Q))$
- (d)  $\neg(P \wedge \neg Q) \vee (P \wedge Q)$
- (e)  $(P \vee Q) \wedge (P \wedge \neg Q)$
- (f)  $\neg(\neg P \wedge Q) \wedge (\neg P \wedge Q)$
- (g)  $(\neg Q \wedge (P \vee R)) \vee (P \wedge Q)$

6. Write the truth set of each of the following statements. Be as explicit as you can, and give complete justification for your answers (note: you may need to use calculus).

- (a)  $n$  is an integer and  $n^2 < 5$
- (b)  $n$  is a positive integer and  $n$  is divisible by 2 or 5
- (c)  $x$  is a real number and  $\sin^2 x = 1$
- (d)  $x$  is a real number and  $\sin x = 0$  and  $\sin 2x = 1$
- (e)  $x$  is a real number and  $x^2 - 6x + 1 = 0$
- (f)  $x$  is a real number and  $x^4 - x^3 + \frac{27}{256} = 0$

7. Evaluate each of the following arguments with truth tables, conclude whether each argument is valid or not, and explain your conclusion.

- (a)
  - Alex is friendly, and Bob or Clara is friendly.
  - Bob is friendly, and Alex or Clara is friendly.
  - Therefore, Clara is friendly.
- (b)
  - Andy and Bin are both tall, or neither of them is.
  - Either Andy is tall or Bin is.
  - Therefore, Bin is tall.