Homework 7 - Math 300 C - Spring 2015 - Dr. Matthew Conroy
Relevant reading: Velleman 7.1, 7.2

1. Prove the following theorems about rational and irrational numbers.
   (a) The sum of an irrational number and a rational number is an irrational number.
   (b) The product of an irrational number and a non-zero rational number is an irrational number.
   (c) The sum of two irrational numbers may be rational (unlike the two theorems above, here an example is sufficient since the theorem is equivalent to the statement that there exists at least two irrational numbers whose sum is rational)
2. Prove that $\sqrt{2}+\sqrt{7}$ is irrational (hint: suppose it is rational, and show that this implies that $\sqrt{2}$ is rational).
3. For each of the following pairs of sets, give a bijection from the first set to the second set. Prove that your bijection *is* a bijection. Then give the inverse of each bijection.
   (a) $\mathbb{Z}$ and $\mathbb{Z} \setminus \{-6, 5\}$
   (b) The set of positive integers congruent to 3 modulo 7 and the set of positive integers congruent to 11 modulo 19
   (c) $(-2, \infty)$ and $(-\infty, 7)$ (these are intervals of real numbers)
   (d) $(-\infty, 3)$ and $(0, 1)$ (these are intervals of real numbers)
4. Let A and B be finite sets. If $A \cap B = \varnothing$, then $|A \cup B| = |A| + |B|$.
5. Let A be a finite set. Prove that if $f : A \to A$ is injective, then f is bijective.
6. Prove that, if $A \sim B$, then $\mathcal{P}(A) \sim \mathcal{P}(B)$.