Homework 6 - Math 300 C - Spring 2015 - Dr. Matthew Conroy

1. Let a and b be integers. Let $f : \mathbb{Z} \to \mathbb{Z}$ be defined by $f(x) = ax + b$. Determine the conditions on a and b necessary and sufficient for f to be a bijection, and then use those conditions to complete, and then prove, the following statement: "f is a bijection iff ________________".

2. Let $A = \{0, 1, 2, 3, 4\}$. Define $f : A \to A$ by $f(x) = 2x + 3$ using modulo 5 arithmetic (so, $f(1) = 0$,$f(2) = 2$, etc.). Define $g : A \to A$ by $g(x) = 3x + 1$ using modulo 5 arithmetic. Prove that $g = f^{-1}$.

3. Let $A = \{0, 1, 2, 3, 4\}$. Define $f : A \to A$ by $f(x) = 4x + 1$ using modulo 5 arithmetic. State and prove a theorem about f^{-1}.

4. Let $f : A \to B$ and $g : B \to A$. Suppose $g \circ f = i_A$. Then f is one-to-one and g is onto.

5. Define $f : \mathbb{Z} \to \mathbb{Z}$ by
$$f(x) = \begin{cases} x + 3 & \text{if } x \text{ is even,} \\ 7 - x & \text{if } x \text{ is odd.} \end{cases}$$
Prove that f is a bijection and give f^{-1}.

6. Let A and B be sets, and $f : A \to B$. Suppose f is one-to-one. Prove that there exists a subset $C \subseteq B$ such that $f^{-1} : C \to A$.