Homework 3 - Math 300 C Spring 2015 - Dr. Matthew Conroy

Prove each of the following theorems. Be sure to use the "theorem/proof" format.

1. Let $\mathcal{A}$ and $\mathcal{B}$ be families (i.e., sets of sets). Then $\cup(\mathcal{A} \cup \mathcal{B}) = (\cup\mathcal{A}) \cup (\cup\mathcal{B})$.
2. Let $\mathcal{A}$ and $\mathcal{B}$ be families. Then $\cup(\mathcal{A} \cap \mathcal{B}) \subseteq (\cup\mathcal{A}) \cap (\cup\mathcal{B})$.
3. There exist non-empty families $\mathcal{A}$ and $\mathcal{B}$ such that $\cup(\mathcal{A} \cap \mathcal{B}) = (\cup\mathcal{A}) \cap (\cup\mathcal{B})$.
4. There exist non-empty families $\mathcal{A}$ and $\mathcal{B}$ such that $\cup(\mathcal{A} \cap \mathcal{B}) \neq (\cup\mathcal{A}) \cap (\cup\mathcal{B})$.
5. Suppose $\mathcal{A}$ and $\mathcal{B}$ are families of sets. Then $(\cup\mathcal{A}) \setminus (\cup\mathcal{B}) \subseteq \cup(\mathcal{A} \setminus \mathcal{B})$.

   Use induction to prove the following theorems.

6. For all integers $n \geq 8$, $n! > 7n^4$.
7. For all integers $n \geq 12$, $6^n > 7(5^n + 4^n)$.
8. Let $a \in \mathbb{R}_{\neq 0}$. For all integers $n \geq 0$,

$$\sum_{i=0}^{n} a^i = \frac{1 - a^{n+1}}{1 - a}.$$

9. Let a and b be positive integers, with $a > b$. For all integers $n \geq 1$, $a - b$ divides $a^n - b^n$.