

Homework 2 - Math 300 C Spring 2015 - Dr. Matthew Conroy

1. DeMorgan's laws and contrapositives

DeMorgan's laws codify the fact that the negation of a statement of the form "A and B" is "not A or not B", and the negation of a statement of the form "A or B" is "not A and not B".

Use DeMorgan's laws to write useful contrapositives of the following sentences.

- (a) If x and y are real numbers, then $x + y$ is a real number.
- (b) If xy is even, then x is even or y is even.
- (c) If you earned at least 90% in my class, then you got an A.
- (d) If it rains or snows, then I will go for a walk but I will not ride my bike.

Prove each of the following theorems. Use a Theorem/Proof format for each one.

- 2. Let x and y be integers. Then $4|x^2 + y^2$ iff x and y are even.
- 3. Let A and B be sets. Then $A \setminus (A \cap B) = A \setminus B$.
- 4. Let A , B and C be sets. Then $(A \setminus C) \cap (B \setminus C) = (A \cap B) \setminus C$.
- 5. Let A , B , and C be sets. Then $(A \cup C) \cap B \subseteq A \cup (B \cap C)$.
- 6. Let A , B , and C be sets. Suppose $A \cup C \subseteq B \cup C$. Then $A \setminus C \subseteq B$.
- 7. Let a , b and c be integers, with $c \neq 0$. Then $a \mid b$ iff $ca \mid cb$.