Homework 7 - Math 301 A - Spring 2014 - Dr. Matthew Conroy

1. Prove that $2x^2 + 3y^2 = z^2$ has no solutions with $x, y, z \in \mathbb{Z}$ and $xyz \neq 0$.

2. Prove that $x^2 + 2y^2 = 5z^2$ has no solutions with $x, y, z \in \mathbb{Z}$ and $xyz \neq 0$.

3. Notice that, for example, $6 = 1+2+3$, $27 = 2+3+4+5+6+7$, $52 = 3+4+5+6+7+8+9+10$, and $195 = 97+98$. What positive integers can be written as a sum of two or more consecutive positive integers? State and prove a theorem that answers this question.

4. The numbers in the sequence $0, 1, 1, 3, 5, 11, 21, 43, 85, \ldots$ defined by

$$j_0 = 0, j_1 = 1, j_n = j_{n-1} + 2j_{n-2} \text{ for } n > 1$$

   are known as the Jacobsthal numbers (A001045). Assuming that $\lim\limits_{n\to\infty} \frac{j_n}{j_{n-1}}$ exists, find the value of this limit.

5. The numbers in the sequence $0, 1, 2, 5, 12, 29, 70, 169, \ldots$ defined by

$$p_0 = 0, p_1 = 1, p_n = 2p_{n-1} + p_{n-2} \text{ for } n > 1$$

   are known as the Pell numbers (A000129). Assuming that $\lim\limits_{n\to\infty} \frac{p_n}{p_{n-1}}$ exists, find the value of this limit.

6. Use a generating function to find the general term of the sequence defined by

$$a_0 = 0, a_n = 3a_{n-1} + 5 \text{ for } n \geq 1.$$