

Homework 6 - Math 301 A - Spring 2014 - Dr. Matthew Conroy

You should read Harold, sections 5.3, 5.4.

1. Let (a, b, c) be a primitive Pythagorean triple. Prove that exactly one of a and b is divisible by 3.
2. Let (a, b, c) be a primitive Pythagorean triple. Prove that exactly one of a , b , and c is divisible by 5. Conclude, using this, the previous problem, and a result proved in class, that

$$60 \mid abc.$$

3. Prove that there are infinitely many Pythagorean triples (a, b, c) with $a < b < c$ and

$$|c - b| = 1.$$

4. Prove that, for all odd $n \geq 3$, there is a Pythagorean triple (a, b, c) with $a = n$ or $b = n$.
5. Prove that, for all primes $p > 5$, there is a Pythagorean triple (a, b, c) with $p \nmid abc$ (i.e., p does not divide a , b or c).