Homework 5 - Math 301 A - Spring 2014 - Dr. Matthew Conroy
You should read Harold, sections 3.3, 3.4, 3.5, 3.7.

1. Prove that $\varphi(n)$ is even unless $n = 1$ or $n = 2$.
2. Prove that there are infinitely many n such that $\varphi(n) \equiv 0 \pmod 4$ and there are infinitely many n such that $\varphi(n) \equiv 2 \pmod 4$.
3. Prove that, for any $d > 1$, there exists infinitely many n such that $d|\varphi(n)$.
4. Prove that 1387 is a pseudoprime.
5. A **reduced residue system mod** n is a set S of n distinct integers such that for all $0 \le j < n$, there exists an $s \in S$ such that $s \equiv j \pmod n$. Show that if g is a primitive root mod n, then
$$\{g, g^2, g^3, \ldots, g^{\varphi(n)}\}$$
is a reduced residue system mod n.
6. Find, directly, the largest number that cannot be written as a sum of positive multiples of 5 and 7. Prove that this number is the largest without relying on any significant theorems (i.e., just use some basic congruence ideas).
7. Do the same as in the previous problem, but with positive multiples of 4, 7 and 11.