Homework 3 - Math 301 A - Spring 2014 - Dr. Matthew Conroy
You should read Harold, sections 3.1, 3.2 and 3.3.

1. Show that if $n \equiv 7 \pmod 8$, then n cannot be expressed as the sum of three squares.
2. Find all solutions to $x^2 \equiv x \pmod{100}$ with as little computation as you can. Do the same with $x^2 \equiv x \pmod{1000}$ similarly.
3. Find the last (i.e., right-most) two digits of 17^{43211} using modular arithmetic.
4. Prove that if a prime is the sum of the squares of three different primes, then one of the squared primes is 3. (For example, $419 = 3^2 + 7^2 + 19^2$ is prime. The first bunch of such primes are listed as sequence A182479 at the Online Encyclopedia of Integer Sequences, oeis.org. Hint: Use congruences modulo 3.)
5. Prove that, for all $n \geq 2$, $12 | 16^n + 10^n - 8$.
6. (You might want to do this problem first.) Let $a, b, c, m \in \mathbb{Z}, m > 0$. Prove that:
   - $a \equiv a \pmod m$
   - If $a \equiv b \pmod m$, then $b \equiv a \pmod m$.
   - If $a \equiv b \pmod m$ and $b \equiv c \pmod m$, then $a \equiv c \pmod m$.
7. Solve the following congruences.
   (a) $4x \equiv 7 \pmod{20}$
   (b) $5x \equiv 6 \pmod{1234}$
   (c) $124x \equiv 7 \pmod{321}$
   (d) $20x \equiv 35 \pmod{855}$
   (e) $76x \equiv 76 \pmod{100}$