1. Define a function \( f : \mathbb{R} \to \mathbb{R} \) by
\[
f(x) = \begin{cases} 
2x & \text{if } x \in \mathbb{Q} \\
-3x & \text{if } x \not\in \mathbb{Q}
\end{cases}
\]
Is \( f \) one-to-one? Is \( f \) onto? Is \( f^{-1} \) a function? State and prove a theorem.

2. Let \( a, b, c \) and \( d \) be real numbers. Suppose \( cd \neq 0 \) and \( ad - bc \neq 0 \).
Define \( f : \mathbb{R} \setminus \{-\frac{d}{c}\} \to \mathbb{R} \setminus \{\frac{a}{c}\} \) by
\[
f(x) = \frac{ax + b}{cx + d}.
\]
(a) Show that \( f \) is one-to-one and onto.
(b) Give a formula for \( f^{-1}(x) \).

3. Let \( A, B \) and \( C \) be sets. Let \( f : A \to B \) and \( g : B \to C \).
(a) Prove that if \( f \) and \( g \) are onto, then \( g \circ f \) is onto.
(b) Prove that if \( g \circ f \) is onto, then \( g \) is onto.
(c) If \( g \circ f \) is onto, is \( f \) necessarily onto? Prove your answer.

4. Let \( A \) be the set of subsets of \( \mathbb{R} \). Define a function \( f : \mathbb{R} \to A \) by
\[
f(x) = \{ z \in \mathbb{R} : |z| > x \}.
\]
Is \( f \) one-to-one? Is \( f \) onto?

5. Let \( A \) and \( B \) be sets, and \( f : A \to B \). Suppose \( f \) is one-to-one. Prove that there exists a subset \( C \subseteq B \) such that \( f^{-1} : C \to A \).

6. For each of the following pairs of sets, give a bijection from the first set to the second set.
(a) \( \mathbb{Z} \) and \( \mathbb{Z} \setminus \{-6, 0, 5\} \)
(b) \( (-2, \infty) \) and \( (-\infty, 7) \)
(c) \( (-\infty, 3) \) and \( (0, 1) \)