1. Prove that, for all \( x \in \mathbb{Z} \), if \( x^2 - 1 \) is divisible by 8, then \( x \) is odd.

2. Prove or give a counterexample for each of the following statements.
   
   (a) For all real numbers \( x \) and \( y \), \( |x + y| = |x| + |y| \).
   
   (b) For all real numbers \( x \) and \( y \), \( |xy| = |x||y| \).
   
   (c) There is a positive integer \( M \) such that, for every positive integer \( n > M \), \( \frac{1}{n} < 0.002 \).
   
   (d) For all integers \( a \) and \( b \), if \( a \mid b \) and \( b \mid a \), then \( a = b \) or \( a = -b \).
   
   (e) For all integers \( m \) and \( n \), if \( n + m \) is odd, then \( n \neq m \).

3. (a) Let \( x \) be an integer. Prove that if \( \sqrt{2}x \) is an integer, then \( x \) is even.
   
   (b) Is the converse of the statement you proved in (a) true? Prove it or give a counterexample.
   
   (c) What can you conclude about \( \sqrt{2}x \) if \( x \) is odd?

4. (a) Suppose \( B \) is a set and \( \mathcal{F} \) is a family of sets. If \( \bigcup \mathcal{F} \subseteq B \) then \( \mathcal{F} \subseteq \mathcal{P}(B) \).
   
   (b) Suppose \( \mathcal{F} \) and \( \mathcal{G} \) are nonempty families of sets. Suppose every element of \( \mathcal{F} \) is a subset of every element of \( \mathcal{G} \). Then \( \bigcup \mathcal{F} \subseteq \bigcap \mathcal{G} \).

5. Define a relation \( T \) on the set \( \mathbb{R} \) of real numbers by
   
   \[ T = \{(x, y) \in \mathbb{R} \times \mathbb{R} : |x - y| < 1\} \]
   
   Is \( T \) an equivalence relation? (Justify your answer, of course.)

6. Define a relation \( R \) on \( \mathbb{Z} \) by
   
   \( (x, y) \in R \iff x - y \) is even.
   
   Determine whether or not \( R \) is reflexive, symmetric and transitive. Is \( R \) an equivalence relation? If \( R \) is an equivalence relation, describe its equivalence classes.

7. Define a relation \( R \) on \( \mathbb{Z} \) by
   
   \( (x, y) \in R \iff xy \equiv 0 \pmod{4} \).
   
   Determine whether or not \( R \) is reflexive, symmetric and transitive. Is \( R \) an equivalence relation? If \( R \) is an equivalence relation, describe its equivalence classes.

8. Let \( A \) be the set of all real functions \( f : \mathbb{R} \to \mathbb{R} \). Define a relation \( R \) on \( A \) by:
   
   \( (f, g) \in R \iff \text{there exists a real constant } k \text{ such that } f(x) = g(x) + k \text{ for all } x \in \mathbb{R} \).
   
   Prove that \( R \) is an equivalence relation.

9. Define a relation \( R \) on \( \mathbb{R} \) by:
   
   \( (x, y) \in R \iff |x - y| < 1 \)
   
   Prove that \( R \) is not an equivalence relation.

10. Let \( A \) and \( B \) be sets. Prove that \( \mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B) \).

11. Let \( m \in \mathbb{Z} \) and suppose \( m > 1 \). Suppose \( a, b, c \in \mathbb{Z} \).

   Prove that if \( a \equiv b \pmod{m} \), then \( ac \equiv bc \pmod{m} \).

12. Prove that if \( n \) is an integer, then \( n^2 \equiv 0, 1, \text{ or } 4 \pmod{8} \).