Homework 4 - Math 300 D Autumn 2014 - Dr. Matthew Conroy
Relevant readings: Velleman, sections 3.3, 3.4, 3.5, and 3.6.

1. Let \( a, b, c \) and \( d \) be integers, with \( bd \neq 0 \). Then \( a\sqrt{b} + c\sqrt{d} \) is an algebraic number.

2. Let \( a \) and \( b \) be integers. Then \( a^2b + a + b \) is even if and only if \( a \) and \( b \) are both even.

3. (a) Let \( n \) be an integer. Then the remainder when \( n^2 \) is divided by 4 is 0 or 1.
   (b) The numbers in the set \{99, 999, 9999, \ldots\} cannot be written as the sum of two squared integers, but at least one can be expressed as the sum of three squared integers.

4. Let \( A \) and \( B \) be sets. Then \( A \subseteq B \) iff \( \mathcal{P}(A) \subseteq \mathcal{P}(B) \).

5. Let \( A \) and \( B \) be sets. Then \( \mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(\mathcal{A} \cup \mathcal{B}) \), with equality if and only if \( A \subseteq B \) or \( B \subseteq A \).

6. Suppose \( \mathcal{R} \) and \( \mathcal{S} \) are families of sets. If \( \mathcal{R} \subseteq \mathcal{S} \), then \( \cup \mathcal{R} \subseteq \cup \mathcal{S} \).

7. Suppose \( \mathcal{R} \) and \( \mathcal{S} \) are families of sets, and \( \mathcal{R} \neq \emptyset \) and \( \mathcal{S} \neq \emptyset \). If \( \mathcal{R} \subseteq \mathcal{S} \), then \( \cap \mathcal{S} \subseteq \cap \mathcal{R} \).

8. Suppose \( \mathcal{R} \) and \( \mathcal{S} \) are families of sets. Then \((\cup \mathcal{R}) \setminus (\cup \mathcal{S}) \subseteq \cup (\mathcal{R} \setminus \mathcal{S}) \).