1. Let $a$ and $b$ be negative real numbers. Prove that if $a < b$ then $a^2 > b^2$.

2. Let $a$, $b$ and $c$ be positive integers. Prove that if $a|b$ and $b|c$, then $a|c$.

3. Let $a$, $b$, and $c$ be integers, $c \neq 0$. If $ac|bc$, then $a|b$.

4. One fact we use all the time when writing proofs is that, if $A \rightarrow B$ and $B \rightarrow C$, then $A \rightarrow C$. Prove this is valid by showing that

$((A \rightarrow B) \land (B \rightarrow C)) \rightarrow (A \rightarrow C)$

is a tautology. Do this by using applicable laws, or a truth table, to show that this is equivalent to a statement which we know is a tautology.

5. Now that we know the irrational numbers exist, we should prove a few facts about them.

You can use the following useful facts in your proofs. You do not have to prove them.

Fact 1: The sum of rational numbers $x=a/b$ and $y=c/d$ is $(ad+bc)/(bd)$.

Fact 2: If $a$ is rational, then $-a$ is rational; if $a$ is irrational, then $-a$ is irrational.

Prove the following theorems:

(a) The sum of two rational numbers is a rational number.
(b) The sum of a rational number and an irrational number is an irrational number.
(c) The product of an irrational number and a non-zero rational number is an irrational number.
(d) The sum of two irrational numbers may be a rational number.