Math 120 Mechanical skills summary

The following are some of the mechanical skills necessary to do well in Math 120, and are worth thinking about for the final exam. Feel free to let me know if you think I've left anything off and I will add them.

- Unit conversion: "multiply by one" method

- Distance formula: find the distance between two points.

- lines

    - Find the equation of the line through two given points.
    - Find the equation of a line given its slope and one point on the line.
    - Find the equation of a line perpendicular to a given line through a given point.
    - Find the point on a line that is closest to a given point not on the line.

- circles

    - Find the equation of a circle given its center and radius.
    - Find the center and radius of a circle given its equation (in any form).
    - Find the equation of a line, tangent to a given circle, which passes through a given point.
    - Find the points of intersection of a line and a circle (how does this process differ when the line is vertical, horizontal, or neither?)

- Uniform linear motion

    - Find the parametric equations of motion of an object exhibiting uniform linear motion:
        * Given the start point and another point together with its corresponding time.
        * Given two points and two corresponding times.
        * Given the start point, another point, and the speed.
        * Given the start point, the line of motion, and the speed and direction.

- Areas

    - Area of a triangle (with a vertical or horizontal side) specified by three points in the plane with area$= \frac{1}{2}bh$ formula.
    - Area of a trapezoid given lengths of two parallel sides and distance between them
    - Area of a rectangle
    - Area of a circle
    - Area of a circular sector given the radius and angle

- Multipart functions

    - Express $|x|$ as a multipart function and use this to express any function of the form $|f(x)|$ as a multipart function.
    - Solve equations involving multipart functions.

- Quadratic functions

    - Find the quadratic function specified by three points.
    - Find the quadratic function specified by its vertex and one other point.

- Find the vertex of a given quadratic function. Determine whether this is the highest or lowest point on the graph of this function.
- Find the maximum and minimum values of a quadratic function on a specified interval.

- Composition/Inverses/Shifting

  - Given $f(x)$ and $g(x)$, determine and simplify the rule for $f(g(x))$.
  - Given a one-to-one function $f(x)$, find $f^{-1}(x)$.
  - Given a function's graph, determine whether or not the function has an inverse.
  - Given a one-to-one function's graph, sketch the graph of $f^{-1}(x)$.
  - Given a quadratic function, restrict its domain to get a one-to-one function and find its inverse (there are two such).
  - Given the graph of a function $f(x)$, sketch the graph of $Af(Bx + C) + D$ for given values, A, B, C, and D.

- Exponential functions

  - Find the exponential function specified by its value at two times.
  - Find the exponential function specified by its value at one time and its doubling time.
  - Find the exponential function specified by its value at one time and its "growth rate" (e.g., 4.5% per year).
  - Solve an equation involving exponential functions (specifically, ones of the form $Ab^t = Cd^t$).
  - Find the doubling time of a given exponential function.

- Linear-to-Linear rational functions

  - Find the linear-to-linear function specified by three points.
  - Find the linear-to-linear function specified by two points and an asymptote.
  - Find the linear-to-linear function specified by a point and two asymptotes.
  - Find the inverse of a given linear-to-linear function.

- Trigonometry

  - Use trigonometric ratios to find the length of sides of right triangles.

- Uniform circular motion

  Specify the parametric equations of motion $x = r\cos(\theta_0 \pm \omega t), y = r\sin(\theta_0 \pm \omega t)$ of an object exhibiting uniform linear motion given:

  - radius r, initial angle θ, direction of motion, angular speed ω
  - radius r, initial angle θ, direction of motion, linear speed v
  - initial angle θ, direction of motion, angular speed ω, linear speed v

- Sinusoidal functions

  - Graph a given sinusoidal function $f(t) = A\sin\left(\frac{2\pi}{B}(t - C)\right) + D$.
  - Determine the sinusoidal function specified by, e.g.:
    * A maximum (or minimum) and the next minimum (or maximum)
    * A maximum (or minimum) and the period

- Solve an equation of the form $A \sin\left(\frac{2\pi}{B}(t - C)\right) + D = k$ and find:
  * The Principal solution
  * The Symmetry solution
  * All other solutions (in particular, as many other solutions as needed)