Modular arithmetic problems + functions problems

1. What digits can an integer square end in? State and prove a theorem.

2. Show that

$$13|6^{123123} - 8$$

3. Show that

$$49|5 \cdot 3^{4m+2} + 53 \cdot 2^{5m}$$

for all integers $m \geq 0$.

4. Let A be a nonempty set. Suppose $f : A \to \mathbb{R}$. Define the relation R by

$$R = \{(a, b) \in A \times A : f(a) = f(b)\}.$$

Show that R is an equivalence relation. Describe the equivalence classes of R.

5. Let A be a nonempty set. Define R by

$$R = \{(x, x) : x \in A\}.$$

Show that R is the only relation on A that is an equivalence relation and a function from A to A.

6. Let A, B and C be sets. Suppose $f : A \to C$ and $g : B \to C$. Prove that if A and B are disjoint, then $f \cup g : A \cup B \to C$.

7. Let $\mathcal{F}$ be the set of all functions from $\mathbb{R}$ to $\mathbb{R}$.

Let $R = \{(f, g) \in \mathcal{F} \times \mathcal{F} : \exists a \in \mathbb{R} \forall x > a(f(x) = g(x))\}$.

   (a) Give an example of a pair of distinct functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ such that $(f, g) \in R$.

   (b) Show that R is an equivalence relation.