Induction

1. Find the smallest $k \in \mathbb{Z}$ such that $n! > n^3$ for all $n \geq k$. Prove the result using induction.

2. Use induction to prove that
$$10|4^{2m+1} + 6^{2m+1}$$
for all $m \in \mathbb{Z}_{\geq 0}$.

3. Let n be a positive odd integer.

   Use induction to prove that the sum of all positive odd integers less than or equal to n is $\left(\frac{n+1}{2}\right)^2$.

4. Let A be a finite set. Prove that if $f : A \to A$ is injective, then f is bijective.

5. Prove that the set of all integer squares is denumerable.

6. Prove that, if $A \sim B$, then $\mathcal{P}(A) \sim \mathcal{P}(B)$.

7. Suppose A is an infinite set and B is a finite subset of A. Prove that $A \setminus B$ is infinite.