Problems on relations

1. Let $A = \mathbb{R}$. Define a relation R on A by

$$(x, y) \in R \Leftrightarrow x < y$$

   Determine whether or not R is an equivalence relation. Prove your answer.

2. Let $A = \mathbb{R}$. Define a relation R on A by

$$(x, y) \in R \Leftrightarrow x \leq y$$

   Determine whether or not R is an equivalence relation. Prove your answer.

3. Let $A = \mathbb{R}$.
   Define a relation R by
$$(a, b) \in R \Leftrightarrow a - b \in \mathbb{Q}.$$

   Is R an equivalence relation? Prove your answer.
   If R is an equivalence relation, describe the equivalence classes of R.

4. How many equivalence relations are there on a set with three elements? List them.

5. Let $A = \mathbb{R} \times \mathbb{R}$.
   Define a relation R on A by

   $((x_1, y_1), (x_2, y_2)) \in R \Leftrightarrow$ the distance from (x_1, y_1) to (x_2, y_2) is a rational number.

   Determine whether or not R is an equivalence relation. Prove your answer.

6. Let $a, b \in \mathbb{Z}$. Let $m \in \mathbb{Z}_{>0}$.
   We say a is **congruent** to b mod m iff $m | (a - b)$.
   If a is congruent to b mod m, we write

$$a \equiv b \pmod{m}.$$

   Show that the relation R on $\mathbb{Z}$ defined by

$$R = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} : a \equiv b \pmod{m}\}$$

   is an equivalence relation.