

Homework 4 - Math 300 B, C Spring 2013 - Dr. Matthew Conroy

Relevant readings: Velleman, sections 3.3, 3.4, 3.5, and 3.6.

1. Let  $A$  and  $B$  be sets. Then  $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$ , with equality if and only if  $A \subseteq B$  or  $B \subseteq A$ .
2. Let  $a$  and  $b$  be integers. Then  $a^2b + a + b$  is even if and only if  $a$  and  $b$  are both even.
3. The numbers in the set  $\{99, 999, 9999, \dots\}$  cannot be written as the sum of two squared integers, but at least one can be expressed as the sum of three squared integers.
4. Let  $a, b$ , and  $c$  be integers,  $c \neq 0$ . If  $ac|bc$ , then  $a|b$ .
5. Let  $A$  and  $B$  be sets. Then  $A \subseteq B$  iff  $\mathcal{P}(A) \subseteq \mathcal{P}(B)$ .
6. Suppose  $\mathcal{R}$  and  $\mathcal{S}$  are families of sets. If  $\mathcal{R} \subseteq \mathcal{S}$ , then  $\cup \mathcal{R} \subseteq \cup \mathcal{S}$ .
7. Suppose  $\mathcal{R}$  and  $\mathcal{S}$  are families of sets, and  $\mathcal{R} \neq \emptyset$  and  $\mathcal{S} \neq \emptyset$ . If  $\mathcal{R} \subseteq \mathcal{S}$ , then  $\cap \mathcal{S} \subseteq \cap \mathcal{R}$ .
8. Suppose  $\mathcal{R}$  and  $\mathcal{S}$  are families of sets. Then  $(\cup \mathcal{R}) \setminus (\cup \mathcal{S}) \subseteq \cup(\mathcal{R} \setminus \mathcal{S})$ .