Homework 3 - Math 300 B, C Spring 2013 - Dr. Matthew Conroy
Relevant readings: Velleman, sections 2.3, 3.1, and 3.2.

1. Let A and B be sets. Prove that $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$.

2. Prove that there exist sets A and B such that $\mathcal{P}(A \cup B) \neq \mathcal{P}(A) \cup \mathcal{P}(B)$.

3. Let a and b be negative real numbers. Prove that if $a < b$ then $a^2 > b^2$.

4. Let a, b and c be positive integers. Prove that if $a|b$ and $b|c$, then $a|c$.

5. One fact we use all the time when writing proofs is that, if $A \to B$ and $B \to C$, then $A \to C$. Prove this is valid by showing that

$$((A \to B) \wedge (B \to C)) \to (A \to C)$$

is a tautology. Do this by using applicable laws to show that this is equivalent to a statement which we know is a tautology.

6. Prove the following theorems:

    (a) The sum of two rational numbers is a rational number.

    (b) The sum of a rational number and an irrational number is an irrational number.

    (c) The product of an irrational number and a non-zero rational number is an irrational number.

    (d) The sum of two irrational numbers may be a rational number.