Homework 1 - Math 300 B, C Spring 2013 - Dr. Matthew Conroy
6 problems

1. Introduce variables and express the following sentences symbolically.

   (a) I will go to London or Paris, but not both.
   (b) Either Dave, Jing or Maria is lying, or they are all lying.
   (c) Peter and Xia are not both over two meters tall.
   (d) Peter and Xia are both not over two meters tall.
   (e) Olga is rich and famous, or she is not rich.
   (f) There is no cake left and I am hungry, or there is cake left but I am not hungry.

2. Write grammatical english sentences with the following structures.

   (a) $(P \wedge Q) \vee \neg P$
   (b) $(A \vee B) \wedge (\neg A \wedge B)$
   (c) $\neg(D \wedge F) \vee F$
   (d) $\neg(G \vee H) \wedge G$

3. Make truth tables of each of the following.

   (a) $P \wedge \neg Q$
   (b) $(P \wedge Q) \vee \neg P$
   (c) $(P \vee Q) \wedge (\neg P \wedge Q)$
   (d) $(P \vee Q) \wedge (P \vee R)$

4. Create truth tables for each of the following. Are any of them equivalent?

   (a) $A \wedge \neg B$
   (b) $\neg(A \vee \neg B)$
   (c) $(A \wedge B) \vee (\neg A \wedge B)$
   (d) $\neg A \vee (A \wedge \neg B)$
   (e) $(A \vee B) \vee (A \wedge \neg B)$

5. Simplify the following expressions.

   (a) $\neg(P \wedge \neg Q) \vee (P \wedge Q)$
   (b) $(P \vee Q) \wedge (P \wedge \neg Q)$
   (c) $\neg(\neg P \wedge Q) \wedge (\neg P \wedge Q)$
   (d) $(P \vee Q) \vee (\neg P \wedge Q)$ ¡— this is redundant, too much like (b) and (a)
   (e) $(\neg Q \wedge (P \vee R)) \vee (P \wedge Q)$

6. Write the truth set of each of the following statements. Be as explicit as you can, and give justification for your answers.

    (a) n is an integer and $n^2 < 5$

    (b) n is a positive integer and n is divisible by 2 or 5

    (c) x is a real number and $\sin^2 x = 1$

    (d) x is a real number and $\sin x = 0$ and $\sin 2x = 1$

    (e) x is a real number and $x^2 - 6x + 1 = 0$

    (f) x is a real number and $x^4 - x^3 + \frac{27}{256} = 0$