MATH 300 D
Final Exam
June 8, 2011

Name ______________________________

Student ID #____________________

HONOR STATEMENT

"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

SIGNATURE:______________________________

1	12	
2	8	
3	6	
4	6	
5	8	
6	10	
Total	50	

- Your exam should consist of this cover sheet, followed by 6 problems. Check that you have a complete exam.
- You are allowed to use only the lists of axioms, elementary properties, and proved results I provide. All other sources are forbidden.
- The use of headphones/earbuds is forbidden during this exam.
- Turn your cell phone OFF and put it AWAY for the duration of the exam.

GOOD LUCK!

1. (12 points) Prove or disprove:

   (a) If $A \subseteq B$ and A is denumerable, then B is denumerable.

   (b) If $A \subseteq B$ and B is denumerable, then A is denumerable.

   (c) If A and B are denumerable, then $A - B$ is denumerable.

   (d) If A and B are denumerable, then $A \cap B$ is denumerable.

   (e) $\mathbb{Q} - \mathbb{N}$ is countably infinite.

2. (8 points)

   (a) Let T be the relation on $\mathbb{R}$ defined by

   $$x\,T\,y \Leftrightarrow |x| = y.$$

   Prove that T is not an equivalence relation.

   (b) Let A be the set of all functions $f : \mathbb{R} \to \mathbb{R}$. Define a relation R on A by:

   $$f\,R\,g \Leftrightarrow \text{ there exists a real constant } c \text{ such that } f(x) = g(x) + c \text{ for all } x \in \mathbb{R}.$$

   Prove that R is an equivalence relation.

3. (6 points) Let A, B, and C be sets and suppose $f : A \to B$ and $g : B \to C$.

   (a) Prove that, if f and g are one-to-one, then $g \circ f : A \to C$ is one-to-one.

   (b) Prove that, if $g \circ f : A \to C$ is onto, then g is onto.

4. (6 points) Prove that $7|(2^{n+2} + 3^{2n+1})$ for every $n \in \mathbb{N}$.

5. (8 points) Define a function $f : \mathbb{R} \to \mathbb{R}$ by:

$$f(x) = \begin{cases} -4x - 30 & \text{if } x < -5 \\ 2x & \text{if } -5 \leq x \leq 5 \\ 15 - x & \text{if } x > 5 \end{cases}$$

Prove that f is surjective but not injective.

6. (10 points) Let A and B be sets, X and Y be subsets of A, and U and V be subsets of B. Suppose $f : A \to B$.

   Recall: If $C \subseteq A$, then $f(C) = \{b \in B : b = f(a) \text{ for some } a \in C\}$. If $D \subseteq B$, then $f^{-1}(D) = \{a \in A : f(a) \in D\}$.

   (a) Prove that $f^{-1}(U) - f^{-1}(V) = f^{-1}(U - V)$.

   (b) Prove that $f(X) - f(Y) \subseteq f(X - Y)$.

   (c) Give an example to show that $f(X - Y)$ need not be a subset of $f(X) - f(Y)$.