MATH 111 - Review for Final Exam

First third of the course:

- Given a table or a graph of a quantity, you should be able to define and compute overall rates of change and incremental rates of change of that quantity.
- Given a graph of a rate of change of a quantity, you should be able to answer questions about that quantity. (Like the Problems from Worksheet 4)
- You should be able to give examples of an overall rate of change and an incremental rate of change.
- Given the graph of a function, you should be able to use the "rolling ruler method" to answer questions about overall and incremental rates of change.
- You should be able to translate between English, the language of graphs, and functional notation. In particular, you should be able to talk about the different rates of change in all three languages.
- Given graphs of TC and TR, you should be able to determine information about profit.
- You should be able to explain the concepts of MR and MC in your own words.
- You should be able to determine the effects MR and MC have on profit.
- Given the graph of TR, you should be able to find values of MR and AR.
- Given the graph of TC, you should be able to find values of MC, AC, and VC. You should also be able to determine the breakeven price. You could also use the graph of TC to determine the shutdown price (how?).
- Given the graph of VC, you should be able to find values of MC and AVC. If you know FC, you should also be able to find values of TC. Also, given VC, you should be able to determine the shutdown price.
- Given the graphs of MC and AC, you should be able to determine the breakeven price. Given the graphs of MC and AVC, you should be able to determine the shutdown price.

Second third of the course:

- Be able to find a linear formula from a verbal description. Phrases to watch for:
  - "...is a linear function of..."
  - "...has a straight-line graph..."
  - "Items sell for \$0.15 each." (Tells you TR is linear with slope 0.15.)
  - "It costs you \$10 to make each Item." (Tells you TC is linear with slope 10.)

- "... [something] changes at a constant rate c..." (Tells you that [something] is linear with slope c.)

- Be able to graph:
  - lines (indicating slope and intercepts)
  - parabolas (indicating vertex and intercepts)

- Remember this HOT TIP: If you are asked when some function is increasing/decreasing/highest/lowest/maximized/minimized/greatest/least, sketch a rough graph of that function and use the graph to answer the question.

- Given the formula for a function $f(x)$, be able to find formulas for:
  - the slope of the diagonal line through $f(x)$: $\dfrac{f(x)}{x}$
    Examples:
    * (average trip speed at time t) $= \frac{D(t)}{t}$
    * $AC(q) = \frac{TC(q)}{q}$
    * $AVC(q) = \frac{VC(q)}{q}$
    * $AR(q) = \frac{TR(q)}{q}$
    * overall rates of change
  - the slope of a secant line through $f(x)$: $\dfrac{f(x+h)-f(x)}{h}$
    Examples:
    * (average speed from t to $t+h$) = (average speed over the h-minute interval starting at t) = $\dfrac{D(t+h)-D(t)}{h}$
    * $MR(q) = \dfrac{TR(q+1)-TR(q)}{1}$ (how does this formula change if q is measured in hundreds or thousands of Items?)
    * $MC(q) = \dfrac{TC(q+1)-TC(q)}{1}$ (how does this formula change if q is measured in hundreds or thousands of Items?)
    * incremental rates of change
  - the area of the rectangle function: $A(x) = x \cdot f(x)$ (WS 14)
    Examples:
    * area of rectangle under price per item function gives $TR(q)$
    * area of rectangle under $AC(q)$ gives $TC(q)$

- Be able to deal with quadratics.

- **READ** questions carefully and completely.

- Remember that the vertex has two coordinates and the vertex formula only gives one of those.

  Some typical questions:

  * Find the time/quantity at which (some quadratic function) is largest/smallest/highest/lowest/maximized/minimized. (Give the "x"-coordinate of the vertex.)
  * Find the largest/smallest/highest/lowest/maximum/minimum value of (some quadratic function). (Give the "y"-coordinate of the vertex.)
  * Find the largest interval on which (some quadratic function) is increasing/decreasing. (Sketch the graph of the parabola to answer the question.)

- Use the quadratic formula to *solve equations*.

- You should understand that the break even price is:
  - the smallest value of $AC(q)$; (So, if AC is a quadratic function of q, then the break even price is the "y"-coordinate of its vertex.)
  - the "y"-coordinate of the point where AC and MC intersect.

- You should understand that the shut down price is:
  - the smallest value of $AVC(q)$; (So, if AVC is a quadratic function of q, then the shut down price is the "y"-coordinate of its vertex.)
  - the "y"-coordinate of the point where AVC and MC intersect.

- Be able to find the quantity that maximizes profit. Either:
  - directly from the formula for profit: $P(q) = TR(q) - TC(q)$. If $P(q)$ is a quadratic whose graph is a parabola that opens downward, then profit is maximized at the q-coordinate of the vertex.
  - using marginal analysis: set $MR = MC$ and solve for q. If you get two positive quantities at which $MR = MC$, then remember that profit is maximized at the transition from $MR > MC$ to $MR < MC$.

- Be able to take a linear formula for instantaneous speed and get formulas for distance using the method developed in Worksheet 16. Be able to go backwards from MR and MC to TR and TC using formulas like those in Worksheet 16, #16 and 18.

Last third of the course:

- Sequences
  - know how to determine if a given sequence is additive, multiplicative or neither
  - be able to find increments, multipliers, recursive formulas, explicit formulas

- know how to find percentage change and proportionate change between any two terms of a sequence (NOTE: Although you can compute the proportionate change between any two terms of a sequence, the **proportionate change of the sequence** is defined to be the proportionate change between consecutive terms. If you're dealing with a multiplicative sequence with multiplier m, then the *proportionate change of the sequence* is $p = m - 1$.)
- multiplicative sequences are particularly useful in bacteria/population problems and in developing the CAF

- Compounding
  - know all the formulas: CAF, $B(k) = N \cdot m^k$, simple interest, discrete compounding, continuous compounding
  - know when and how to use all the formulas
  - be able to solve problems involving present and future values (don't get confused by the wording!)
  - understand what APY is, when to use it, and be able to compute it for discrete compounding and continuous compounding
  - be able to solve equations by using the natural logarithm, $\ln$
  - in discrete compounding, be able to solve for **any** variable, except n

$$A = P\left(1 + \frac{r}{n}\right)^{nt} \text{ solving for } t \text{ requires using } \ln$$

  - in continuous compounding, be able to solve for **any** variable

$$A = Pe^{rt} \text{ solving for } r \text{ or } t \text{ requires using } \ln$$