Summary for Midterm One - Math 125

Here are some thoughts I was having while considering what to put on the first midterm. The core of your studying should be the assigned homework problems: make sure you really understand those well before moving on to other things (like old midterms).

- 4.10 - Antiderivatives
  - You should know what it means for $f(x)$ to be an antiderivative of $g(x)$.
  - Given two functions, $f(x)$ and $g(x)$, you should be able to say whether or not $f(x)$ is an antiderivative of $g(x)$.
  - How many antiderivative does a function have?
  - What is that "$+C$" business all about?
- 5.1 - Areas and Distances
  - How can we approximate the area of a region in the plane?
  - What is an interpretation of the area under the graph of a velocity function?
- 5.2 - The Definite Integral
  - You should understand the definition of the *definite integral* and its relation to *area* under a curve.
  - You should be able to use the *midpoint* rule to approximate a definite integral.
  - Problems 35-40 are particularly nice.
- 5.3 - The Fundamental Theorem of Calculus
  - Part 1: If f is continuous on $[a, b]$, then

    $$g(x) = \int_a^x f(t)\, dt$$

    is continuous on $[a, b]$ and differtiable on (a, b), $g'(x) = f(x)$.
  - Part 2: If f is continuous on $[a, b]$, then

    $$\int_a^b f(x)\, dx = F(b) - F(a)$$

    where F is any antiderivative of f.
  - Be sure you can differentiate functions like

    $$g(x) = \int_{\sin x}^{x^3} e^{t^2}\, dt$$

    using the chain rule and part 1 of the FTOC (see, e.g., problems 50-52).

- 5.4 - Indefinite Integrals and the Net Change Theorem
  - Here we get the notation that
    $$\int f(x)\,dx$$
    stands for the most general antiderivate of f.
- 5.5 - The Substitution Rule
  - The substitution rule is the most important and powerful tool for finding antiderivatives. It can be considered, to a certain extent, the reverse of the chain rule for differentiation.
  - Substitution is a way of getting from one indefinite integral to another. When trying to find antiderivatives, we may need to try several different substitutions until hitting on one that improves the integral we are working with to the point that we can find the antiderivative. Sometimes, more than one substitution, used in sequence, is an effective way to go. Practice will improve your ability to *see* the right substitutions.
  - As we get more techniques for finding antiderivatives, the substitution method will always be with us. It will pay to make sure you can use the method well now.
  - There are *tons* of practice problems in this section to work on to improve your substitution ability (e.g., problems 7 - 44, and 49 - 70).
- 6.1 - Areas between Curves
  - The first of our many applications of the integral is to find the area between curves.
  - "Area is the integral of width"
  - In many instances you will want to express the area as an integral in y rather than in x.
  - It very often helps to have a decent sketch of the region whose are you are trying to find.
  - Note that
    $$\int_a^b (f(x) - g(x))\ dx = -\int_a^b (g(x) - f(x))\ dx$$
    so if your answer comes out negative (which is impossible for an area) check that you haven't got the difference of the two functions in the wrong order.
- 6.2 - Volumes (by Cross Section)
  - Here is developed the idea that "volume is the integral of (cross sectional) area".

  - Although many of the examples we looked at involve *solids of revolution* whose cross-sections are circles, this method applies to any solid that has cross sections whose area can be expressed as a function of x (or y).

- 6.3 - Volume by Cylindrical Shells
  - The method of washers/disks is great, but in certain cases we can result in an integral we are unable to evaluate, or, indeed, to setup. So we have another method: the method of cylindrical shells.
  - Even if the washer/disk method works, the cylindrical shells method can be easier. Practice will help you decide which method to use on a given solid. Problems 7-14 on page 469 are good for this.