Review for the Second Midterm Exam - Math 124

The primary topics for this exam are:

- **derivatives, including implicit differentiation**
- **related rates**
- **linear approximation and Newton's method**
- **maxima and minima, inflection points and curve sketching**

The relevant sections of the text are 3.4-3.6, 3.9, 3.10, 4.1, and 4.3. Additionally, you should understand how to use Newton's Method (see, e.g., Tuesday's worksheet and section 4.8 of the textbook).

Here are some things to consider about each topic.

Derivatives

For this exam, you should know how to differentiate everything. That is, you should be able to find $f'(x)$ given $f(x)$, and $f(x)$ could be any combination of algebraic expressions, trigonometric functions, logarithmic functions and exponential functions.

Also, you should be able to find $\frac{dy}{dx}$ given an implicit equation which relates y and x. Generally, this means using *implicit differentiation*.

You should understand and be able to use the technique of *logarithmic differentiation*.

There are tons of problems in Stewart to practice. For starters, problems 7-54 of section 3.4, 5-36 of section 3.5, and problems 2-50 of 3.6 cover all the basic procedures.

Also, problems 1-50 on page 262 can't be beat.

From 3.5, I also like 59-62, 65 and 66.

Related Rates

Related rates problems come in a pretty wide variety; every problem in section 3.9 would be good practice. Try to work a bunch of different ones, particularly from the later problems in the section.

Problems 91-101 on page 264 are also related rates problems.

Linear Approximation and Newton's Method

Sections 3.10 and Newton's Method are closely related: they both illustrate how the tangent line to the graph of a function can be used to estimate the value of the function.

Problems 23-31 of section 3.10 are good, basic practice, as are the assigned non-textbook problems.

For Newton's method, you should be able to find the root of any not too complicated equation. Problems 61-63 on page 349 are good practice, as are problems 13-28 from section 4.8, though the later ones may require some work to get good starting values.

Maxima and Minima, Increasing and Decreasing, and Inflection Points

You should know how to:

- Determine the maximum and minimum values of a function on a closed interval
- Classify local extrema of a function using the first derivative or second derivative test
- Determine where a function is increasing and where it is decreasing
- Determine where a function's graph is concave up and where it is concave down
- Determine the inflection points of a given function

Keep in mind that the second derivative test can fail: sometimes you have to use the first derivative test. Can you give an example of when the second derivative test fails?

There are lots of good problems to practice all of the necessary techniques. A few good ones are found in section 4.5. Try problems 4.5 #5, 12, 13, 28, 39, 47. Don't worry about making a sketch: just collect all the necessary information: critical numbers, increasing/decreasing intervals, local extrema, concavity and inflection points.