Review for the Final Exam - Math 124K

The primary topics for this exam are:

- **derivatives**
- **limits (including L'Hospital's rule)**
- **related rates**
- **linear approximation and Newton's method**
- **maxima and minima, inflection points and curve sketching**

Here are some things to consider about each topic.

Derivatives

You should know how to differentiate everything. That is, you should be able to find $f'(x)$ given $f(x)$, and $f(x)$ could be any combination of algebraic expressions, trigonometric functions, logarithmic functions and exponential functions.

Also, you should be able to find $\frac{dy}{dx}$ given an equation which relates y and x. Generally this means using *implicit differentiation*.

You should be able to determine $\frac{dy}{dx}$ of any point on a curve defined parametrically by, say, $x = f(t)$ and $y = g(t)$.

You should understand and be able to use the technique of *logarithmic differentiation*.

There are tons of problems in Stewart to practice. For starters, problems 7-54 of section 3.4, 5-36 of section 3.5, and problems 2-50 of 3.6 cover all the basic procedures.

Also, problems 1-50 on page 262 can't be beat.

From 3.5, problems 5-20 are good basic practice. Problems 65 and 66 are good, too. From 3.6, number 54 is fun now that you know L'Hospital's rule.

Limits and L'Hospital's Rule

You should be able to evaluate many sorts of limits.

It may be worth paying particular attention to practice non-L'Hospital techniques, as L'Hospital's rule can make us forget that sometimes other methods are needed. Think: what do I do if L'Hospital's rule does not help?

Problems 15-36 of section 2.6 and 3-28 on page 167 are all good practice.

Problems 5-64 of section 4.4 are all good, too.

Some things to remember:

- L'Hospital's rule and the quotient rule are different things.
- You should check and **indicate** that the application of L'Hospital's rule is valid. Indicate this by writing $\frac{0}{0}$ or $\frac{\infty}{\infty}$ above the equal sign where the rule has been applied.
- You may need to apply L'Hospital's rule more than once to get a result. (Can you give an example of when this happens?)
- In some cases, even if L'Hospital's rule is applicable, another method may be simpler.

Related Rates

Related rates problems come in a pretty wide variety; every problem in section 3.9 would be good practice. Try to work a bunch of different ones, particularly from the later problems in the section.

Problems 97-101 on page 264 are also related rates problems.

Linear Approximation and Newton's Method

Sections 3.10 and 4.8 are closely related: they both illustrate how the tangent line to the graph of a function can be used to estimate the value of the function.

Problems 23-31 of section 3.10 are good, basic practice, as are the assigned non-textbook problems.

For Newton's method, you should be able to find the root of any not too complicated equation. You can easily create your own problems, and then check your answers by plugging in your result.

Problems 13-28 from section 4.8 are all good, though the later ones may require some work to get good starting values.

Maxima and Minima, Inflection Points, Curve Sketching and Optimization

You should know how to:

- Determine the maximum and minimum values of a function on a closed interval
- Classify local extrema using the first derivative or second derivative test
- Determine the inflection points of a given function
- Sketch a curve, incorporating information about local extrema, increasing/decreasing regions, inflection points, and asymptotes
- Use all of these techniques to solve optimization problems

Keep in mind that the second derivative test can fail: sometimes you have to use the first derivative test. Can you give an example of when the second derivative test fails?

There are lots of good problems to practice all of the necessary techniques. A few good ones are found in section 4.5. Try problems 4.5 #5, 12, 13, 28, 39, 47.

To determine asymptotes, you may need L'Hospital's rule.

Optimization problems are a lot like related rates problems in that the setup is critical: the work you do before applying calculus is often the most the challenging part. Be sure to practice plenty of these kinds of problems. All of the problems in section 4.7 are good practice, but there are way too many to do all of them. Try a sampling. The problems that give you the function to be optimized will not give you practice with setting up such a function, so those problems are not as good for practice. Be sure to include some problems that involve trigonometry (46 is a classic), as these often have a different feel.

Keep in mind when solving optimization problems that it is never sufficient to just find a critical point. You must give calculations supporting your claim that this point yields a maximum, or minimum. Usually either the first-derivative test or second-derivative test is needed.