Review Problems for Exam I
MATH 126 – Spring 2008

Here are some problems from old exams that will help you prepare for the midterm. This is not intended to be an exhaustive review of the material. You will be expected to understand how to do all the assigned homework. I suggest you work these problems only after re-working most or all of the homework.

1. Give the first four non-zero terms of the Taylor Series expansion for
$$f(x) = \sin x$$
based at $\frac{\pi}{4}$.

2. The Taylor series for $g(x)$ based at $b = 0$ is
$$1 - 9x + 16x^2 - 25x^3 + \ldots$$
What is $g^{(3)}(0)$?

3. Let $f(x) = e^x \sin x$.
   (a) Find the second-order Taylor polynomial $T_2(x)$ for $f(x)$ based at $b = 0$.
   (b) Give a bound on the error $|f(x) - T_2(x)|$ for x in the interval $-0.1 \leq x \leq 0.1$.

4. Find the first four non-zero terms of the Taylor series for
$$f(x) = xe^{x^2} - \frac{1}{4 + x^2}$$
based at $b = 0$.

5. Let $f(x) = 3 - 7x - 4x^2 + 6x^3$.
   (a) Find the 2nd-degree Taylor polynomial $T_2(x)$ for $f(x)$ based at $b = 1$.
   (b) Use the Quadratic Approximation Error Estimate to give an upper bound on the error $|f(x) - T_2(x)|$ on the interval $(0.75, 1.25)$.

6. Give the coefficient on x^{11} in the Taylor series for $f(x) = x^3 e^{x^2}$ based at $b = 0$.

7. Find the 2nd-degree Taylor polynomial, $T_2(x)$ for the function $f(x) = \ln(\ln x)$ based at $x = e$.

8. Approximate the integral
$$\int_0^2 \sin(x^2)\, dx$$
by using the first four non-zero terms of a Taylor series. Give a decimal approximation of your result.

9. Write out the first four terms of the Taylor series for the function
$$f(x) = \frac{1}{1 + 5x} + \frac{1}{3 + x}$$

10. Find the length of the curve
$$x = 4\sqrt{t},\ y = \frac{t^3}{3} + \frac{1}{2t^2},\ 1 \leq t \leq 4.$$

11. Does the curve defined by the polar equation
$$r = \sec\theta + \tan\theta$$
intersect the vertical line $x = 2$? Explain.

12. Find the slope of the tangent line to the polar curve

$$r = \frac{1}{\theta}, \theta > 0$$

at the point where it intersects the cartesian curve

$$x^2 + y^2 = \frac{1}{9}.$$

13. Find all points of intersection between the curve defined by the polar equation

$$r = 4 \csc \theta$$

and the line $y = x$.

14. Find all values of x such that $\langle 4, 5, x \rangle$ and $\langle 3x, 7, x \rangle$ are orthogonal.

15. Find a vector that is orthogonal to the vector $\langle 11, 3, -5 \rangle$ and has length 7.

16. Find the angle between the vectors $\vec{a} =< -3, 4, 1 >$ and $\vec{b} =< 3, 1, 1 >$. Give a decimal value for the angle.

17. Suppose the vector $< x, 3, 2 >$ is orthogonal to the vector $< 2, 3, x >$. Find x.

18. The set of points that are twice as far from the origin as they are from the point $(5, 5, 5)$ is a sphere. Find its center and radius.

19. At what point(s) is the tangent line to the curve

$$x = t^3 - 3t, y = t^2 + 2t$$

parallel to the line with parametric equations

$$x = 3t + 5, y = t - 6 \text{ ?}$$

20. Eliminate the parameter in the following parametric equation pair to get a Cartesian equation for the curve that involves no trigonometric functions.

$$x = \cos t, y = \sin t - \cos t$$

21. Consider the curve defined parametrically by the parametric equations

$$x = \ln \ln t, y = \ln t - (\ln t)^2.$$

Find the equation of the tangent line to the curve at the point $t = e$.