Review Problems for Exam II
MATH 126 – Spring 2008

Here are some problems from old exams that will help you prepare for the midterm. This is not intended to be an exhaustive review of the material. You will be expected to understand how to do all the assigned homework. I suggest you work these problems only after re-working most or all of the homework.

1. Find the parametric equations for the line that is the intersection of the plane $x + y + 3z = 8$ with the plane $2x - y - z = 4$.

2. Find the parametric equations for the line that is the intersection of the plane $x + y + 2z = 1$ and the plane $3x - y + 4z = 1$.

3. Find the equation of the plane containing the line

$$x = 3 + t, y = 4 - 2t, z = 1 - t$$

and the point $(1, 2, 5)$.

4. Find the equation of the plane that passes through the point $(3, -1, 2)$ and contains the line

$$x = 5 - t, y = 3 + 3t, z = 8 + 2t.$$

5. Find the equation of the plane containing the line of intersection of the two planes

$$x + y + z + 5 = 0 \text{ and } 3x + 2y - z + 2 = 0$$

and the point $(1, 2, 1)$.

6. Find the equation of the plane containing the line of intersection of the two planes

$$x + y - z = 3 \quad \text{and} \quad 2x - 3y + 4z = 5$$

and the point $(-2, 7, 3)$.

7. Find the equation of the plane through the three points $(-3, 4, 0)$, $(1, 7, -3)$ and $(2, -5, 3)$.

8. Find the point of intersection of the two lines

$$x = 4 - t, y = 6 + 2t, z = -1 + 3t \text{ and } x = 1 + 2t, y = 14 - 8t, z = 7 - 4t.$$

9. Let S be the surface defined as the set of points p (in three-dimensional space) such that the distance from p to the plane $y = 5$ equals the distance from p to the line

$$y = 1, z = 2.$$

   (a) Find an equation for S.

   (b) Find the equation of the trace of S in the plane $z = 6$. Describe the trace (i.e. what kind of curve is it?).

10. Consider the curve defined by the vector equation

$$\overrightarrow{r}(t) = \langle 4t, 5t^3, 2t^2 \rangle$$

   (a) Find the unit tangent vector $\overrightarrow{T}(t)$ at the point where $t = 1$.

   (b) Find the parametric equations of the tangent line the curve at the point where $t = 1$.

11. At what point(s) is the tangent line to the curve

$$x = t^3 - 3t, y = t^2 + 2t$$

parallel to the line with parametric equations

$$x = 3t + 5, y = t - 6 \text{ ?}$$

12. Consider the space curve defined by the vector function

$$\mathbf{r}(t) = \langle t^2 - 3, 2e^{t-6}, 4 - t^3 \rangle.$$

Find all values of t such that a tangent vector to $\mathbf{r}(t)$ is parallel to the line

$$x = 3 + 4t, y = \frac{4}{3} + \frac{2}{3}t, z = -36t.$$

13. Find a vector function that represents the intersection of the cylinder $x^2 + y^2 = 9$ and the plane $x + z = 2$.

14. Consider the curve in the xy-plane defined by the position vector function

$$\vec{r}(t) = \langle t^2 - 3t, t^2 + 2t \rangle$$

Find the t-value of the point of maximum curvature on this curve.

15. For any $m > 0$, the helix determined by the position function

$$\vec{r}(t) = \langle \cos t, \sin t, mt \rangle$$

has constant curvature that depends on m. Find the value of m such that the radius of curvature at any point on the curve is 3.

16. Reparametrize the curve

$$\vec{r}(t) = \langle 5t - 1, 2t, 3t + 2 \rangle$$

with respect to arc length measured from the point where $t = 0$ in the direction of increasing t.

17. Suppose a particle is moving in 3-dimensional space so that its position vector is

$$\vec{r}(t) = \langle t, t^2, \frac{1}{t} \rangle.$$

   (a) Find the tangential component of the particle's acceleration vector at time $t = 1$.
   (b) Find all values of t at which the particle's velocity vector is orthogonal to the particle's acceleration vector.

18. A particle is moving so that its position is given by the vector function

$$\vec{r}(t) = \langle t^2, t, 5t \rangle$$

Find the tangent and normal components of the particle's acceleration vector.

19. Let $f(x, y) = xe^y - \ln(x + y)$.

   (a) Sketch the domain of f.
   (b) Find $f_{xy}(x, y)$.

20. Let $f(x, y) = x^2y + x \sin y - \ln(x - y^2)$.

   (a) Find $f_y(x, y)$.
   (b) Find $f_{xy}(x, y)$.

21. Let $f(x, y) = x^4y^3 - 3xy^2 + 4x^5 - \frac{6}{y^2} + (e^{x^3 - x})(\ln y)$. Compute f_x, f_{xx}, and f_{xy}.

22. Consider the function

$$f(x, y) = \sqrt{4 + 2x^2 - 3y^2}.$$

   (a) Describe and graph the level set of f of level $c = 2$.
   (b) Find an equation of the tangent plane to the surface $z = f(x, y)$ at the point $(2, 1, 3)$.
   (c) Use the linear approximation to approximate $f(1.9, 1.2)$.

23. Let

$$f(x, y) = \frac{2x^2 + y^2}{\ln(2x - y)}.$$

(a) Find and sketch the domain of f.

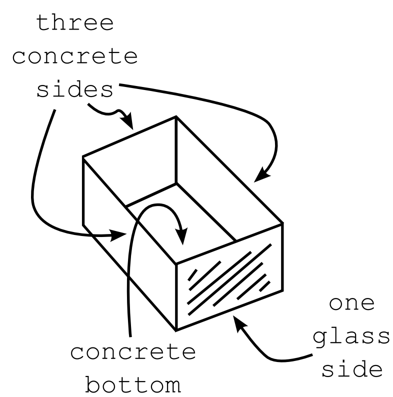
(b) Consider the surface $z = f(x, y)$. Find the equation of the tangent plane to the surface at a point (x_0, y_0, z_0) with $x_0 = y_0 = e$.

(c) Using the linear approximation at (e, e) estimate $f(3, 3)$.

24. Find the local maximum and minimum values and the saddle points of the function

$$f(x, y) = x^3 - 12x - 6y + y^2 + 1.$$

25. You wish to build a large swimming pool in the shape of a parallelpiped. It will essentially be an open-top box made of concrete. One side, however, will be made of glass, so that the pool can be observed from below ground.



Concrete costs \$15 per square meter, and glass costs \$100 per square meter. If the volume of the pool must be 1000 cubic meters, what should the dimensions be to minimize the cost of the pool?

26. Find and classify all the critical points of the function $f(x, y) = 3xy - x^3 - y^3$.