Review for the Second Midterm Exam - Math 124

The primary topic of this exam is **derivatives** and their applications.

The sections of the text relevant for this exam are 3.1, 3.2, 3.4-3.8, 3.10, 3.11 and 4.1.

Here's a summary of each section.

- 3.1 Derivatives of Polynomials and Exponential Functions

  In this section, the first derivative rules are developed, particularly the **power rule** and the fact that
  $$\frac{d}{dx}e^x = e^x.$$
  I particularly like problems 45, 54, and 59 from this section.

- 3.2 The Product and Quotient Rules

  Two mainstays of differentiation, the product and quotient rules are introduced here.

  I like problems 31, 33, and 41.

- 3.4 Derivatives of Trigonometric Functions

  The key facts from this section are
  $$\frac{d}{dx}\sin x = \cos x$$
  and
  $$\frac{d}{dx}\cos x = -\sin x.$$
  You should know how to find the derivatives of all the other trig functions ($\tan x$, $\sec x$, etc.) using these facts.

  Problem 33's not bad.

- 3.5 The Chain Rule

  The chain rule is the *pièce de résistance* of differentiation, allowing us to extend all the other rules to functions of unlimited complexity.

  The chain rule comes in two forms. The first, and the most commonly used so far in the course, is, in abbreviated form,
  $$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x).$$

The second form, while equivalent, looks a bit different. Again in abbreviated form, it states that

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}.$$

This can be interpreted as stating that derivatives can be treated, to a certain extent, like fractions.

One of the most common applications of the chain rule is in the form of the **generalized power rule**:

$$\frac{d}{dx}(f(x))^n = n(f(x))^{n-1}f'(x).$$

The problems from this section are good practice for applying the rule.

- 3.6 Implicit Differentiation

  With all the derivative rules behind us, we look in this section toward extending our ability to apply them to other situations. In particular, we want to be able to discuss $\frac{dy}{dx}$ when y and x are related by an equation, but not necessarily by a function.

  Here, the key is to remember your differentiation rules, and in particular remember the chain rule, so that the derivative (with respect to x) of y^5 is

  $$5y^4\,\frac{dy}{dx}$$

  and not just

  $$5y^4.$$

  Problems 29, 35, 63 and 69 are good ones.

- 3.7 Higher Derivatives

  A short chapter, whose significance will appear in later courses (and to a limited extent later in this course).

  Problems 25 and 31 are the key ones for the mechanical aspects.

- 3.8 Derivatives of Logarithmic Functions

  The key fact in this section is that

  $$\frac{d}{dx}\ln x = \frac{1}{x}.$$

  From this, we can differentiate any logarithm function, since

  $$\log_b x = \frac{\ln x}{\ln b}.$$

Another idea in this section is that of **logarithmic differentiation**. This means using the natural logarithm, and the logarithm rules, to simplify the work needed to differentiate certain expressions (for instance, we can avoid some applications of the product and quotient rules with this method). See problems 35-46.

- 3.10 Related Rates

  The key idea here is that **if two quantities are related by an equation, we can differentiate the equation with respect to time (t) to get an equation relating their rates of change**.

  The key challenge in these problems can be the process of converting the statement of the problem into an equation relating variables whose rates of change we want to know. So, use the recipe provided in lecture to help you.

  Problems that were not assigned but are good for more practice include 8, 17, 18, 37 and any others that sound interesting to you.

- 3.11 Linear Approximation

  A small topic in this course, but a huge one in the bigger picture, and you'll encounter it in later courses. The big idea: the tangent line is a function in itself, and is a good approximation of the function near the point of tangency.

  Symbolically,

  $$f(x) \sim L(x) = f(a) + f'(a)(x - a) \text{ for } x \sim a$$

  Problems 5-14 in the section are all good practice.

- 4.1 Maximum and Minimum Values

  A simply stated powerful idea: **Continuous functions on closed intervals attain maximum and minimum values, and they attain them at critical points or at endpoints.**

  Critical points are points where the derivative is undefined, or equals zero.

  Problems 47-62 are all good practice.