Math 112 B, C - Winter 2005
Mid-Term Exam Number One Solutions
January 27, 2005

1. *Compute the derivatives. Do not simplify.*

   (a) *Find* $f'(x)$ *if* $f(x) = \frac{3}{5}x - 4x^2 + \sqrt{x} + \frac{1}{6}x^{3/4}$

   $$f'(x) = \frac{3}{5} - 8x + \frac{1}{2}x^{-1/2} + \frac{1}{8}x^{-1/4}.$$

   (b) *Find* $T'(q)$ *if* $T(q) = q\left(\frac{3}{q} - \frac{4}{q^2}\right)$

   $$T'(q) = \frac{4}{q^2}.$$

   (c) *Find* $d'(t)$ *if* $d(t) = \frac{t + 4t^6 - 7t^{12}}{t^2}$

   $$d(t) = t^{-1} + 4t^4 - 7t^{10}$$

   so

   $$d'(t) = -1t^{-2} + 16t^3 - 70t^9.$$

2. *You are in the business of making and selling items. With* q *representing hundreds of items, let* $P(q)$ *be your profit if you make and sell* q *items. You don't have a formula for* $P(q)$*, but instead have a formula for the change in* P *if you change your production level from* q_1 *to* q_2*:*

   $$P(q_2) - P(q_1) = (q_2 - q_1)(12 - q_1 - q_2)$$

   (a) *Find* $P(8) - P(3)$.

   $$P(8) - P(3) = (8 - 3)(12 - 8 - 3) = (5)(1) = 5.$$

   (b) *Which is larger,* $P(5)$ *or* $P(6)$*?*
   Since

   $$P(6) - P(5) = (6 - 5)(12 - 6 - 5) = (1)(1) = 1$$

   we can tell that

   $$P(6) = P(5) + 1$$

   so $P(6)$ is larger.

   (c) *Find the slope of the secant line between the points* $(2, P(2))$ *and* $(2.5, P(2.5))$.
   The slope is

   $$\frac{P(2.5) - P(2)}{2.5 - 2} = \frac{(2.5 - 2)(12 - 2 - 2.5)}{0.5} = 7.5.$$

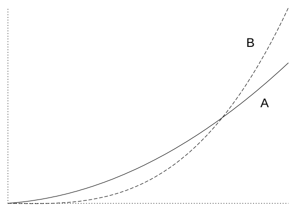
   (d) *Find* $P'(2)$.
   First we calculate

   $$\frac{P(2 + h) - P(2)}{h} = \frac{h(12 - 2 - (2 + h))}{h} = 8 - h.$$

   As h gets closer and closer to zero, $8 - h$ gets closer and closer to 8. Hence

   $$P'(2) = 8.$$

3. *Two rocket cars, A and B, are moving along parallel tracks. The cars start at the same starting point. The distance, in miles, of each car from the starting point over the first ten minutes of a test run is shown in the graph below.*



Each car's distance from the starting point after t minutes is given by:

$$\begin{aligned} A(t) &= 0.6t^2 + t \\ B(t) &= 0.1t^3 + 0.25t \end{aligned}$$

(a) *Find a one minute interval during which car A moves 3 miles.*

We want to find t so that

$$A(t+1) - A(t) = 3$$

That is,

$$0.6(t+1)^2 + (t+1) - 0.6t^2 - t = 3$$

which can be simplified to

$$1.2t + 1.6 = 3$$

Solving for t, we find $t = \frac{7}{6} = 1.166666\ldots$.

(b) *When are the cars moving at the same speed?*

The speed of car A is given by

$$A'(t) = 1.2t + 1$$

and the speed of car B is given by

$$B'(t) = 0.3t^2 + 0.25$$

so they are moving at the same speed if

$$1.2t + 1 = 0.3t^2 + 0.25$$

which can be rearranged to

$$0 = 0.3t^2 - 1.2t - 0.75.$$

Solving for t with the quadratic formula, we find the positive solution is $t = 4.54950975$.

(c) *When is car B traveling twice as fast as its overall average speed?*

Car B's overall average speed is

$$\frac{B(t)}{t} = 0.1t^2 + 0.25$$

so car B will be traveling twice as fast as its overall average speed when

$$0.3t^2 + 0.25 = 2(0.1t^2 + 0.25)$$

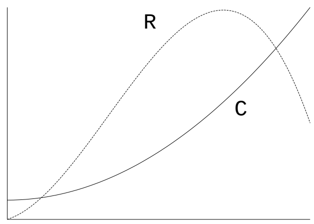
This can be rearranged to

$$t^2 = 2.5$$

so the solution is $t = \sqrt{2.5} = 1.58113883\ldots$.

4. *You are in the business of making and selling ennui detectors. Your total revenue (TR) and total cost (TC), in hundreds of dollars, for producing and selling q hundred detectors is*

$$\begin{aligned} TR : R(q) &= -0.05q^3 + 5q^2 + 50q \\ TC : C(q) &= q^2 + 1000 \end{aligned}$$



(a) *Find $MR(40)$, the slope of the tangent line to the graph of TR at $q = 40$.*

$$MR(40) = R'(40) = -0.15(40)^2 + 10(40) + 50 = 210.$$

(b) *Find a value of q at which MC equals* $\$55$.

$$MC(q) = 2q$$

so if MC equals $\$55$, we must have

$$2q = 55$$

so $q = 27.5$..

(c) Find the value of q that yields maximum profit.
We want to find q where $MC(q) = MR(q)$, i.e., where $C'(q) = R'(q)$. This is

$$2q = -0.15q^2 + 10q + 50$$

$$0 = -0.15q^2 + 8q + 50$$

which has solutions

$$q = -5.651199 \text{ and } q = 58.984532$$

Since only positive q are meaningful, profit must be maximum at $q = 58.984532$.