Step Functions

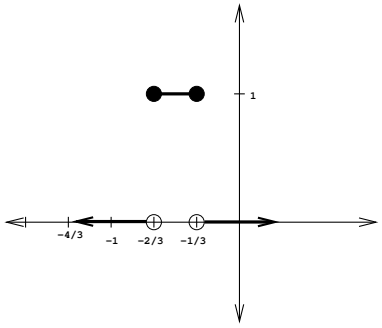
Let $u(t)$ be the basic step function:

$$u(t) = \begin{cases} 0 & \text{if } t < 0, \\ 1 & \text{if } 0 \le t \le 1, \\ 0 & \text{if } t > 1 \end{cases}$$

1. Find the multipart rule for each of the following functions. Sketch the graph of each function.
   (a) $u(3t+2)$
   (b) $\frac{1}{2}u(2t-4)$
   (c) $u(t)+u(t-2)$
   (d) $u(2t-1)-u(\frac{t}{2})$
   (e) $tu(t)$
   (f) $(t+2)u(t-2)$
   (g) $\dfrac{u(2t)+1}{u(t-3)+1}$

Answers:

1. (a) $$u(3t+2) = \begin{cases} 0 & \text{if } t < -\frac{2}{3}, \\ 1 & \text{if } -\frac{2}{3} \le t \le -\frac{1}{3}, \\ 0 & \text{if } t > -\frac{1}{3} \end{cases}$$



(b) $$\frac{1}{2}u(2t-4) = \begin{cases} 0 & \text{if } t < 2, \\ 1 & \text{if } 2 \le t \le 2.5, \\ 0 & \text{if } t > 2.5 \end{cases}$$

(c) $$u(t)+u(t-2) = \begin{cases} 0 & \text{if } t < 0, \\ 1 & \text{if } 0 \le t \le 1, \\ 0 & \text{if } 1 < t < 2, \\ 1 & \text{if } 2 \le t \le 3, \\ 0 & \text{if } t > 3 \end{cases}$$

(d) $$u(2t-1)-u(\frac{t}{2}) = \begin{cases} 0 & \text{if } t < 0, \\ -1 & \text{if } 0 \le t < \frac{1}{2}, \\ 0 & \text{if } \frac{1}{2} \le t \le 1, \\ -1 & \text{if } 1 < t \le 2, \\ 0 & \text{if } t > 2 \end{cases}$$

(e) $$tu(t) = \begin{cases} 0 & \text{if } t < 0, \\ t & \text{if } 0 \le t \le 1, \\ 0 & \text{if } t > 1 \end{cases}$$

(f) $$(t+2)u(t-2) = \begin{cases} 0 & \text{if } t < 2, \\ t+2 & \text{if } 2 \le t \le 3, \\ 0 & \text{if } t > 3 \end{cases}$$

(g) $$\frac{u(2t)+1}{u(t-3)+1} = \begin{cases} 1 & \text{if } t < 0, \\ 2 & \text{if } 0 \le t \le \frac{1}{2}, \\ 1 & \text{if } \frac{1}{2} < t < 3, \\ \frac{1}{2} & \text{if } 3 \le t \le 4, \\ 1 & \text{if } t > 4 \end{cases}$$