Rational Function Sketching Example

$$\text{Graph } h(x) = \frac{2x^2 + 4x - 6}{x^2 + 3x + 2}.$$

- It will make things easier if we factor the numerator and denominator first:

$$h(x) = \frac{2(x+3)(x-1)}{(x+2)(x+1)}.$$

- Find the domain: $D = \{\text{all real } x \neq -2, -1\}$
- Once you've found the domain, reduce to lowest terms: this function is already in lowest terms.
- Find the zeros (x-intercepts): $h(x) = 0$ when $x = -3$ or $x = 1$.
- Find the y-intercept: $h(0) = -3$.
- Find the vertical asymptotes. Once you've reduced to lowest terms, the vertical asymptotes will be at all values of x that make the denominator equal to 0. This function has vertical asymptotes at $x = -2$ and $x = -1$.
- Find the horizontal asymptotes by determining the behavior of the y-values of the function as x gets very, very large and positive or large and negative. (The trick is to multiply the numerator and denominator by one over the highest power of x in the function.)

$$h(x) = \frac{2x^2 + 4x - 6}{x^2 + 3x + 2} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \frac{2 + \frac{4}{x} - \frac{6}{x^2}}{1 + \frac{3}{x} + \frac{2}{x^2}}$$

  As x gets very, very large, the bits with the x's in them get very, very small. So, $h(x)$ heads toward 2 as x heads toward $\pm\infty$. The horizontal asymptote is the line $y = 2$.

- As x goes from $-\infty$ to ∞, the only places where the values of $h(x)$ can change sign are the zeros of $h(x)$ and the vertical asymptotes. Split up the real line at these values of x: $x = -3$ (a zero), $x = -2$ (a vertical asymptote), $x = -1$ (a vertical asymptote) and $x = 1$ (a zero). You have the intervals: $(-\infty, -3)$, $(-3, -2)$, $(-2, -1)$, $(-1, 1)$, and $(1, \infty)$.
- Pick test values in each interval and determine whether $h(x)$ is positive or negative there.

Interval	Test Value	Sign
$(-\infty, -3)$	-5	$h(-5) > 0$
$(-3, -2)$	-2.5	$h(-2.5) < 0$
$(-2, -1)$	-1.5	$h(-1.5) > 0$
$(-1, 1)$	0	$h(0) < 0$
$(1, \infty)$	5	$h(5) > 0$

- Set up your axes. Graph the asymptotes (horizontal and vertical) and the x- and y-intercepts. Use the information in the table to sketch a rough graph of the function.