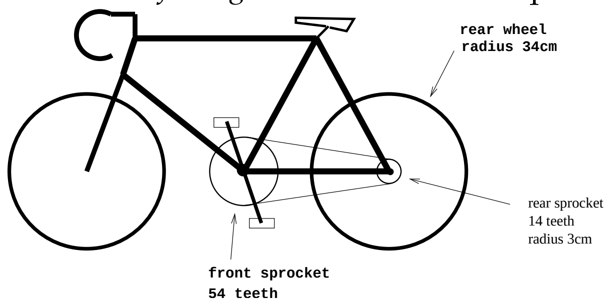
An angular/linear speed bicycle example

On October 1, 2003, Leontien Zijlaand-van Moorsel set a new women's hour record by riding a bicycle 46.065 km in one hour on the velodrome at Mexico City.

She rode a *fixed gear* bike which was qualitatively like this one:



A fixed gear means that there is no freewheel: the rear sprocket is attached directly to the rear wheel, so that if the wheel turns, the rear sprocket (and hence the front sprocket and pedals) turns. You can't "coast" on such a bike. These kinds of bikes are standard in track racing. They also have no brakes, to make it difficult to make sudden speed changes. This *improves* safety in the close quarters of track racing.

Here's the questions: **If Leontien rode at a constant speed, how fast did she pedal?** That is, how quickly must her pedals (and feet) have been going around? In cycling, this rate is known as the *cadence*.

Here's the idea: if we know how fast the wheels turn, then we'll know how fast the rear sprocket turns, then we'll know how fast the chain moves, then we'll know how fast the front sprocket turns, then we'll know how fast the pedals turn.

Any wheel, sprocket, gear, etc., that turns has both an **angular speed** and a **linear speed**:

The angular speed is the rate at which the thing turns, described in units like revolutions per minute, degrees per second, radians per hour, etc.

The linear speed is the speed at which a a point on the edge of the object travels in its circular path around the center of the object. The units can be any usual speed units: meters per second, miles per hour, etc.

If v represents the linear speed of a rotating object, r its radius, and ω its angular velocity in units of radians per unit of time, then

$$v = r\omega.$$

This is an extremely useful formula: it related these three quantities, so that knowing two we can always find the third.

Now, the linear speed of a wheel rolling along the ground is also the speed at which the wheel moves along the ground. So if we assume that Leontien moved at a constant speed, then her wheels were always moving 46.065 km/hr, or

$$46.065\frac{\text{km}}{\text{hr}}\left(\frac{1000\text{m}}{1\text{km}}\right)\left(\frac{1\text{hr}}{3600\text{sec}}\right) = 12.7958\text{m/sec}.$$

This is the linear speed of her wheels.

Since the rear wheel has a radius of $r = 0.34$ meters, the angular speed of the rear wheel is given by

$$\omega = \frac{v}{r} = \frac{12.7958\text{ m/sec}}{0.34\text{ m}} = 37.6347\text{ radians/sec}.$$

Since the rear sprocket is attached directly to the rear wheel, it rotates exactly as the rear wheel does: every revolution of the rear wheel is a revolution of the sprocket. Hence, the angular speed of the rear sprocket is $\omega_{rs} = 37.6347$ radians/sec.

Knowing that the radius of the rear sprocket is 0.03 m, we can calculate the linear speed of the rear sprocket:

$$v_{rs} = \omega_{rs} r_{rs} = (37.6347\text{ radians/sec})(0.03\text{ m}) = 1.12904\text{ m/sec}.$$

Every point in a sprocket-chain system moves at the same linear speed. Hence every point on the chain has a (linear) speed of 1.12904 m/sec, and the front sprocket has a linear speed of $v_{fs} = 1.12904$ m/sec.

We now need the radius of the front sprocket in order to find its angular speed. We can use the fact that the number of teeth on a sprocket must be proportional to its circumference (so, for instance, if we double the circumference of the sprocket, we double the number of teeth). Thus,

$$\frac{54}{r_{fs}} = \frac{15}{r_{rs}} = \frac{15}{0.03\text{ m}}$$

so that

$$r_{fs} = \frac{54(0.03\text{ m})}{15} = 0.108\text{ m}.$$

With this, we calculate the angular speed of the front sprocket:

$$\omega_{fs} = \frac{v_{fs}}{r_{fs}} = \frac{1.12904\text{ m/sec}}{0.108\text{ m}} = 10.4541\text{ rad/sec}.$$

Putting this into more convenient units, we have

$$\omega_{fs} = 1.6638\text{ rev/sec} = 99.829\text{ rev/min} = 99.829\text{ rpm}.$$

So Leontien was pedalling about 100 revolutions per minute.