**Math 111**

Midterm Exam Number Two Review

- Be able to find a linear formula from a verbal description. Look for words like "...is a linear function of...", "...has a straight line graph...", "items sell for \$0.15 each" (tells you TR is linear), "it costs you \$10 to make each item" (tells you TC is linear).

- Given a formula for distance, you should be able to find formulas for average trip speed and average speed over an interval of a specified length (see WS 12). You should be able to do this in other contexts as well.

  e.g., Given the total amount of water in a vat, $W(t)$, find formulas for overall and incremental rates of change. Or, given the TC, find formulas for AC and MC.

- Given a formula for price per item as a function of $q$, you should be able to find formulas for $TR(q)$ and $MR(q)$:

  $TR(q) = p \cdot q$ $\qquad$ $MR(q) = TR(q+1) - TR(q)$ $\qquad$ (see WS 13)

- If you have formulas for $MR$ and $MC$ and you want the quantity that maximizes profit, you should be able to set $MR = MC$ and solve for $q$. Round **up** to the nearest whole number of items (being careful if $q$ is measured in hundreds, e.g., $q = 2.13$ *is* a whole number of items.) If you get two positive quantities at which $MR = MC$, remember profit is maximized at a transition from $MR > MC$ to $MR < MC$.

- You should know when and how to apply the vertex formula/method. Suppose $f(x) = ax^2 + bx + c$ with $a < 0$. Then the vertex formula $x = \frac{-b}{2a}$ gives *the value of* $x$ that maximizes $f(x)$. To find the maximum value of $f(x)$, you can plug $x = \frac{-b}{2a}$ back into $f(x)$.

  Typical questions:

  - Find the time/quantity at which (some quadratic function) is largest/smallest/ lowest/highest/maximized/minimized.
  - Find the largest/smallest/lowest/highest/maximum/minimum value of (some quadratic function).

  Special note: If profit is a quadratic function and you use the vertex formula to maximize profit and $q = \frac{-b}{2a}$ is not a whole number, then you should round to the *nearest* whole number quantity.

- The vertex formula can also give information about when a quadratic function is increasing/decreasing. In particular, if you're asked to find an interval on which a function is increasing, draw a rough sketch.

e.g.:

$$f(x) = -3x^2 + 12x - 19 \text{ opens down, vertex at } x = \frac{-12}{2(-3)} = 2$$

$$\text{so } f(x) \text{ increases from } x = 0 \text{ to } x = 2, \text{ for instance}$$

As opposed to $g(x) = -3x + 4$ which is always decreasing.

- Know when and how to apply the quadratic formula. Use the quadratic formula to solve an equation of the form $ax^2 + bx + c = 0$.

  Example: Suppose profit is given by

  $$P(q) = -3q^2 + 12q - 19.$$

  $P(q)$ is a quadratic function but it is not a quadratic equation. However if we want to know what quantity makes a profit of \$15, then we can solve the quadratic equation

  $$-3q^2 + 12q - 19 = 15$$

  by rewriting it as

  $$-3q^2 + 12q - 34 = 0$$

  and applying the quadratic formula.

- You should understand how TR, TC and profit can be read as areas of rectangles under the price and $AC$ per item functions. (WS 15)

- You should be able to answer questions about $TR$, $TC$, and profit given the formulas for $p$ and $AC$. (WS 15)

- You should have a clear understanding of how all of the following are related:

  - $TR$ and $MR$ and $AR$ and $p$
  - $TR(q) = p \cdot q$, $MR(q) = TR(q+1) - TR(q)$, $AR(q) = \frac{TR(q)}{q} = p$
  - TC, FC, VC, AC, AVC, MC:

$$TC(q) = VC(q) + FC$$

$$FC = TC(0)$$

$$AC(q) = \frac{TC(q)}{q}$$

$$AVC(q) = \frac{VC(q)}{q}$$

$$MC(q) = TC(q+1) - TC(q)$$

- You should understand that **breakeven price** is
  - the smallest value of $AC$ (so if $AC$ is a quadratic function of $q$ this is the "y"-coordinate of the vertex of $AC$)
  - the "y"-coordinate of the point where $AC$ and $MC$ intersect
- You should be able to determine distance travelled from informatino about speed. In particular, if speed is a known linear function of time, then you should be able to describe distance as a quadratic function of time, and answer questions about those functions. (WS 17)