**Solution 1:** First, we find the coordinates of $P$ and $Q$ as functions of $r$. Then we can find the equation of the line determined by these two points, and thus find the $x$-intercept (the point $R$), and take the limit as $r \to 0$.

The coordinates of $P$ are $(0, r)$. The point $Q$ is the point of intersection of the two circles $x^2 + y^2 = r^2$ and $(x - 1)^2 + y^2 = 1$. Eliminating $y$ from these equations, we get $r^2 - x^2 = 1 - (x - 1)^2 \iff r^2 = 1 + 2x - 1 \iff x = \frac{1}{2}r^2$. Substituting back into the equation of the shrinking circle to find the $y$-coordinate, we get

$$(\frac{1}{2}r^2)^2 + y^2 = r^2 \iff y^2 = r^2(1 - \frac{1}{4}r^2) \iff y = r\sqrt{1 - \frac{1}{4}r^2}$$ (the positive $y$-value). So the coordinates of $Q$ are $\left(\frac{1}{2}r^2, r\sqrt{1 - \frac{1}{4}r^2}\right)$. The equation of the line joining $P$ and $Q$ is thus

$$y - r = \frac{r\sqrt{1 - \frac{1}{4}r^2} - r}{\frac{1}{2}r^2 - 0}(x - 0).$$ We set $y = 0$ in order to find the $x$-intercept, and get

$$z = -r\frac{\frac{1}{2}r^2}{\sqrt{1 - \frac{1}{4}r^2} - 1} = -\frac{1}{2}r^2 \left(\frac{\sqrt{1 - \frac{1}{4}r^2} + 1}{1 - \frac{1}{4}r^2 - 1}\right) = 2\left(\sqrt{1 - \frac{1}{4}r^2} + 1\right).$$

Now we take the limit as $r \to 0^+$:

$$\lim_{r \to 0^+} x = \lim_{r \to 0^+} 2\left(\sqrt{1 - \frac{1}{4}r^2} + 1\right) = \lim_{r \to 0^+} 2(\sqrt{1} + 1) = 4.$$

So the limiting position of $R$ is the point $(4, 0)$. 