1. (12 points, 3pts each) Using the derivative rules you have learned, compute the derivatives. You do not need to simplify your final answer. You must BOX YOUR FINAL ANSWER.

(a) \( y = \sin^{-1}(\ln(x^2 + 1)) \),

\[
y' = \frac{1}{\sqrt{1-(\ln(x^2 + 1))^2}} \cdot \frac{1}{x^2 + 1} \cdot 2x
\]

(b) \( y = \sin^2(3t^2 - t + 1) \),

\[
\frac{dy}{dt} = 2\sin(3t^2 - t + 1) \cdot \cos(3t^2 - t + 1) \cdot (6t - 1)
\]
1. (cont.) (c) If \( f(x) = \sqrt{x} + \sqrt{2x} \),
\[
f'(x) = \frac{1}{2 \sqrt{x+\sqrt{2x}}}.
\]
1 pt numerator
1 pt numerator
1 pt numerator
1 pt numerator

(d) \( y = e^{3x} \),
\[
y'(x) = \frac{d}{dx} e^{3x} = 3e^{3x}.
\]
0 pt did this
0 pt get \( y' \)
0 pt part
0 pt get \( y' \)
0 pt part

\[
\ln y = \ln x \ln x = \ln x \cdot \ln x = (\ln x)^2.
\]
0 pt got this
0 pt did this

\[
y' = \frac{y'}{y} = 2 \ln x \cdot \frac{1}{x}.
\]
0 pt got this
0 pt did this

\[
y' = y \cdot \frac{2 \ln x}{x} = \frac{\ln x \cdot 2 \ln x}{x}.
\]
2. (9 points) The graphs of \( f(x) = x^2 + 1 \) and \( g(x) = -x^2 + x \) are pictured below. Find the equation of the pictured line which is simultaneously tangent to both curves.

**Label tangent points:**

**Calculate slope of \( L \) three ways:**

\[
\text{slope } L = \frac{a^2 + 1 - (-b + x)}{a - b} = \frac{a^2 + b^2 - b + 1}{a - b}
\]

**Combine to get:**

\[
-2b + 1 = \frac{(-b + \frac{x}{2})^2 + b^2 - b + 1}{-b + \frac{x}{2} - b} = \frac{b^2 - b + \frac{x}{2} + b^2 - b + 1}{-2b + \frac{x}{2} - b}
\]

**Get a value for \( b \):**

\[
(2b + 1)(-2b + \frac{x}{2}) = 2b^2 - 2b + \frac{x}{4} = 0
\]

**Set \( b \) value equal to 0:**

\[
b = \frac{1 + \sqrt{1 + 4\frac{\frac{x}{4}}{2}}}{2} = 1 + \frac{\sqrt{x}}{2}
\]

**Conclude \( b = \frac{1 - \sqrt{x}}{2} \):**

\[
-2b + 1 = 1 + \frac{\sqrt{x}}{2} + \frac{1}{2} = \frac{1 + \sqrt{x}}{2}
\]

**Equation of the line:**

\[
y = \frac{1 + \sqrt{x}}{2} (x - \frac{1 + \sqrt{x}}{2}) + \frac{(1 + \sqrt{x})^2}{4} + 1
\]

**Note:** The equation is given; no partial credit.
3. (10 points) The graph of the equation $2(x^2 + y^2)^3 = 25(x^2 - y^2)$ is pictured below.

(a) (6pts) Find the equation of the tangent line to the curve at $(3, 1)$.

$2, 2(4x^2y^2 - 2xy^3) = 25(x^2 - y^2)$

End @ $(3, 1)$:

$4(9 + 1)(6 - 2y') = 25(6 - 2y')$

$2y' + 80 = 150 - 50y'$

$130 + 50y' = -90$

$y' = -\frac{170}{50}$

$y' = -\frac{17}{5}$

(b) (4pts) Let $Q = (0, 1.01)$ be the point on the curve in the first quadrant with $y$-coordinate 1.01. Using linear approximation, estimate the value of $a$. Leave your answer in exact form.

$y' = -\frac{9}{13} (x - 3) + 1$

$\frac{x}{100} = -\frac{9}{13} (x - 3)$

$\frac{x}{100} = -\frac{9}{13}$

$3x = 900$

$x = \frac{900}{3}$

$x = 300$
4. [8 points] Sand is being dumped from a conveyor belt at a rate of 2 m³/min and forms a right circular cone. Assume the radius of the cone is always three times as large as the height of the cone. (Recall, the volume of a right circular cone is \( V = \frac{1}{3} \pi r^2 h \), where \( r \) is the radius of the circular base and \( h \) is the height of the cone.) Find the rate at which the height of the cone is increasing when the height is 9 m.

We know \( \frac{dV}{dt} = 2 \); we want \( \frac{dh}{dt} \) when \( h = 9 \).

\[
V = \frac{1}{3} \pi r^2 h \quad \Rightarrow \quad 3h = \sqrt{\frac{3V}{\pi}}
\]

\[
V = \frac{1}{3} \pi (3h)^2 h = 3 \pi h^3 \quad \Rightarrow \quad V = \frac{3 \pi h^3}{3} = \frac{1}{3} \pi r^2 h
\]

\[
\Rightarrow \quad \frac{dV}{dt} = 9 \pi h^2 \frac{dh}{dt}
\]

\[
\frac{dh}{dt} = \frac{dV}{dt} \div (9 \pi h^2)
\]

If \( h = 9 \) and \( \frac{dV}{dt} = 2 \), \( \frac{dh}{dt} = \frac{2}{9 \pi \cdot 9^2} = \frac{2}{729 \pi} \)

(1 pt) Plugged data in correctly

(1 pt) Correct final answer. Error appears fine, answer ok.
5. (11 points) A particle is moving in the xy-plane with parametric equations

\[ x(t) = e^t + e^{-t} \]
\[ y(t) = e^t \]

at time \( t \) seconds, \( t \geq 0 \). The units on each axis are centimeters (cm). Recall that the speed of the particle is given by the formula

\[ s(t) = \sqrt{(x'(t))^2 + (y'(t))^2}. \]

This problem studies the values of the speed on the time interval \([0,1]\).

(a) (3pts) Find \( s(t) \) as an explicit function of \( t \).

\[ s(t) = \sqrt{(e^t - e^{-t})^2 + (e^t + e^{-t})^2} = \sqrt{e^{2t} - 2 + e^{2t} + e^{-2t}} \]

They need not simplify.

(b) (2pts) Calculate \( s(0) \) and \( s(1) \).

\[ s(0) = \sqrt{1 - 2 + 1} = 1 \]
\[ s(1) = \sqrt{e^{2} - 2 + \frac{2}{e^2}} \approx 2.379 \]

(c) (6pts) Find the critical numbers for \( s(t) \) and the minimum speed on the time interval \([0,1]\).

\[ s'(t) = \frac{2e^{2t} - 4e^{-2t}}{2e^{2t} - 2 + 2e^{-2t}} = \frac{e^{2t} - 2 - 2e^{-2t}}{e^{2t} + 2e^{-2t}} \]

\[ s'(t) = 0 \Rightarrow 0 = e^{2t} - 2 + 2e^{-2t} \Rightarrow e^{4t} = \frac{ln^2 2}{4} \approx 0.1173 \]

\[ s\left( \frac{ln 2}{4} \right) = \sqrt{1 - 2 + \frac{2}{e^{ln 2}}} \approx \sqrt{2} \approx 1.414 \]

\[ \text{min speed occurs at time } \frac{ln 2}{4} \text{ and } \approx \sqrt{2} - 2 + \frac{ln 2}{4} \]