5.2 Exponential Modeling

1. In 1968, the U.S. minimum wage was \$1.60 per hour. In 1976, the minimum wage was \$2.30 per hour. Assume the minimum wage grows according to an exponential model $w(t)$, where t represents the time in years after 1960.

   (a) Find a formula for $w(t)$.

   (b) What does the model predict for the minimum wage in 1960?

   (c) If the minimum wage was \$5.15 in 1996, is this above, below or equal to what the model predicts.

2. The town of *Pinedale, Wyo.* is experiencing a population boom. In 1990, the population was 860 and five years later it was 1210.

   (a) Find a linear model $l(x)$ and an exponential model $p(x)$ for the population of Pinedale in the year 1990+x.

   (b) What will be the population of Pinedale in 2000 under these two models?

   (c) Using graphical techniques, discuss when the predicted population of the linear model exceeds that of the exponential model by at least 10.

   (d) Use a graphing device to determine when the predicted population under the linear model exceeds the exponential model population by at least 10.

3. Tiffany is a model rocket enthusiast. She has been working on a pressurized rocket filled with laughing gas. According to her design, if the atmospheric pressure exerted on the rocket is less than 10 pounds/sq.in., the laughing gas chamber inside the rocket will explode. Tiff worked from a formula $p = (14.7)e^{-h/10}$ pounds/sq.in. for the atmospheric pressure h miles above sea level. Assume that the rocket is launched at an angle of α^o above level ground at sea level with an initial speed of 1400 feet/sec.

   (a) If the angle of launch is $\alpha = 12^o$, determine the minimum atmospheric pressure exerted on the rocket during its flight. Will the rocket explode in mid-air?

   (b) If the angle of launch is $\alpha = 82^o$, determine the minimum atmospheric pressure exerted on the rocket during its flight. Will the rocket explode in mid-air?

   (c)* Assume that the maximum elevation to avoid premature explosion is 3.8526 miles. Find the largest launch angle α so that the rocket will not prematurely explode.

4. You have been hired for a job for 4 weeks (20 days), 8 hours each day. You have the option of being paid \$20 per hour or at rate of 1 cent for the first day, tripling each day thereafter (i.e. \$ 0.01 the first day, \$0.03 after two days work, \$0.09 after three days work, \$0.27 after four days work, etc.).

   (a) Assume you pay 32% tax on your pay, what is the take home pay under each payment scheme?

   (b) Certainly, if you only work one day, the better deal is to take the first payment scheme. After how many days work does it become better to adopt the second payment plan?

5. The State of Washington has advertised it will offer be selling "Tahoma Bonds" (T-bonds) to the public. A T-bond of face value B dollars is purchased for $(2/5)B$ dollars. The conditions of the T-bond state that the current cash value after t years is computed using three criteria:

First, for T-bonds held less than 30 years, the current cash value is computed by using yearly compounding and an annual interest rate of $r = 3.45\%$. For T-bonds held exactly 30 years, the current cash value is computed by using continuous compounding and an annual interest rate of $r = 5.45\%$. For T-bonds held more than 30 years, the current cash value is computed by applying quarterly compounding for $t - 30$ years at an annual rate of $r = 8.5\%$ to the 30 year T-bond cash value.

(a) What is the current cash value of a T-bond of face value \$5000 cashed after 18 years?

(b) What is the current cash value of a T-bond of face value \$5000 cashed after 30 years?

(c) What is the current cash value of a T-bond of face value \$5000 cashed after 38 years?

(d) If you desire to have \$100,000 after holding a T-bond of face value B dollars for 45 years, what is B?

6. Given a positive integer n, recall that the notation $n! = n(n-1)(n-2)\dots(3)(2)(1)$, which is called *$n$ factorial*. Most calculators will have a dedicated key for this calculation. The n^{th} Taylor polynomial $\mathcal{T}_n(f)$ for the exponential function $f(x) = e^x$ is given by the formula:

$$\mathcal{T}_n(f) \quad = \quad 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + + \dots + \frac{x^n}{n!}.$$

(a) Write down the first six Taylor polynomials for the exponential function.

(b) Use a graphing device to plot each of these six functions.

(c) Use a. to approximate the value of the number e; compare with your calculator output.

7. Define two new functions:

$$y = \cosh(x) = \frac{e^x + e^{-x}}{2}$$

and

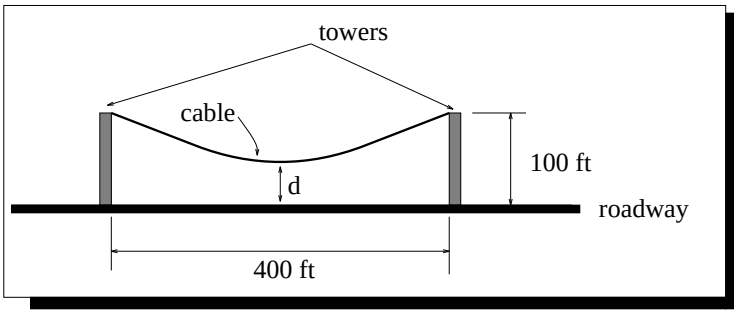
$$y = \sinh(x) = \frac{e^x - e^{-x}}{2}.$$

These are called the basic *hyperbolic trigonometric functions*.

(a) Sketch the graphs of these two functions via a graphing device.

(b) Sketch the graph of the $u^2 - v^2 = 1$ in the uv-coordinate system; this is called the *unit hyperbola*. For any value x, show that the point $P(x) = (\cosh(x), \sinh(x))$ is on the unit hyperbola. (Hint: Verify that $[\cosh(x)]^2 - [\sinh(x)]^2 = 1$, for all x.)

8. A hanging cable is modeled by a portion of the graph of the function $y = a\cosh(\frac{x-h}{a}) + C$, for appropriate constants a, h and C. The constant h depends on how the coordinate system is imposed. A cable for a suspension bridge hangs from two 100 ft. high towers located 400 ft. apart. Impose a coordinate system so that the picture is symmetric about the y-axis and the roadway coincides with the x-axis. The hanging cable constant is $a = 500$ and $h = 0$.



Find the minimum distance from the cable to the road.

9. Return to the Earning Power Problem in §1.4. Use the data in Table 1.4.1 to obtain exponential models $M(x)$ and $W(x)$ for Men's and Women's Earning power in the year $1970 + x$, respectively. What will be the earnings in 1997? In 2010? In 2100? From these calculations, what can you say about whether women are gaining on men?

10. Given the initial information P_o, r and n, Banks will sometimes provide an additional percentage rate called the *effective yield*. This is understood to be an annual interest rate $s\%$ which would yield the same amount at the end of one year under one compounding as the given data under n compoundings.

    (a) If P_o dollars is invested at an annual rate of 7% compounded quarterly, what is the effective yield?

    (b) If P_o dollars is invested at an annual rate of 6.75% continuously compounded, what is the effective yield?

11.*

    (a) For each $r = 1, 2, 3, 4, 5, \ldots$, define a NEW function $T_r(x)$ on the domain [0,1] by the rule $T_r(x) = \cos(r \cos^{-1}(x))$. Use a graphing device to sketch accurate graphs of $T_r(x)$ for $r = 1, \ldots, 6$.

    (b) Use a graphing device to give a convincing argument that the six functions in a. are given by these polynomials (and decide which polynomial goes with each $T_r(x)$).

$$\begin{array}{ll} -1 + 18x^2 - 48x^4 + 32x^6 & -3x + 4x^3 \\ 5x - 20x^3 + 16x^5 & -1 + 2x^2 \\ 1 - 8x^2 + 8x^4 & x \end{array}$$

    (c) Form the NEW function

$$f(x) = 1.266066 + 1.130318T_1(x) + 0.271495T_2(x) + 0.044337T_3(x) + 0.005474T_4(x).$$

    Use b. to write this as a polynomial, then plot $f(x)$ and e^x in a common coordinate system on the domain [0,1]. What is happening?

12. In 1989, research scientists published a model for predicting the cumulative number of AIDS cases reported in the United States:

$$a(t) = 155\left(\frac{t - 1980}{10}\right)^3, \quad \text{(thousands)}$$

where t is the year. This paper was considered a "relief", since there was a fear the correct model would be of exponential type. Pick two data points predicted by the research model $a(t)$ to construct a *new* exponential model $b(t)$ for the number of cumulative AIDS cases. Using a graphing device, sketch $a(t)$ and $b(t)$ simultaneously; discuss how the two models differ and explain the use of the word "relief".

13. Assume that the U.S. Deficit on January 1, 1995 was calculated to be 4 Trillion dollars; i.e. \$4,000,000,000,000. On September 1, 1995 the Deficit was \$5,271,000,000,000. If the Deficit is growing according to an exponential model $d(t)$, where t represents time in days, what is the model? What will be the Deficit on January 1, 1996. Compare the "debt increase/day" on January 1, 1995 and January 1, 1996? (In other words, determine the increase in the deficit from day 0 to day 1, then from day 365 to day 366.)