5.1 Functions of Exponential Type

1. Let's brush up on the required calculator skills. Use a calculator to approximate:
   (a) 3^{π}
   (b) $4^{2+\sqrt{5}}$
   (c) π^{π}
   (d) $5^{-\sqrt{3}}$
   e. 3^{π^2}
   f. $\sqrt{11^{\pi-7}}$
   g. $(5^{\sqrt[3]{23}})\sin(\frac{11}{3}\pi)$
   h. $\sin^{-1}(2^{-\pi})$

2. Using graphical reasoning (and no logarithms) solve these equations:
   (a) $3^x = 3^5$
   (b) $2^x = 0$
   (c) $2^x = 1$
   d. $5^x = 25$
   e. $(3^x - 1)(2x - 5) = 0$
   f. $\pi^x = \sqrt{\pi}$

3. Graphically determine the number of solutions of the equation $1 + 4x - x^2 = 2^x$. Use a graphing device to approximate these solutions.

4. Refer to Figure 5.1.4.
   (a) Find the frequency of middle C.
   (b) Find the frequency of A above middle C.
   (c) What is the frequency of the lowest note on the keyboard? Is there a way to solve this without simply computing the frequency of every key below $A220$?
   (d) The Bosendorfer piano is famous, due in part, to the fact it includes additional keys at the left hand end of the keyboard, extending to the C below the bottom A on a standard keyboard. What is the lowest frequency produced by a Bosendorfer?

5. Sketch the graphs of $f(x) = 3^x$ and $g(x) = 2x + 2$. How many solutions does the equation $f(x) = g(x)$ have? Use a graphing device to find these solutions.

6. Begin with a sketch of the graph of the function $y = 2^x$ on the domain of all real numbers. Describe how to use the "four tools" of §2.5 to obtain the graphs of these functions: $y = -2^x, y = 2^{-x}, y = 3(2^x), y = \frac{1}{3}(2^x), y = 3 + 2^x, y = 2^x - 2, y = 2^{x-2}, y = 2^{x+2}, y = 2^{3x}, y = 2^{x/3}$.

7. Put each equation in standard exponential form:
   (a) $y = 3(2^{-x})$
   (b) $y = 4^{-x/2}$
   (c) $y = \pi^{\pi x}$
   d. $y = 1\left(\frac{1}{3}\right)^{3+\frac{x}{2}}$
   e. $y = \frac{5}{0.345^{2x-7}}$
   f. $y = 4(0.0003467)^{-0.4x+2}$

8. (a) Begin with the function $y = f(x) = 2^x$.

   i. Rewrite each of the following functions in standard exponential form:

$$f(2x), f(x-1), f(2x-1), f(2(x-1)), 3f(x), 3f(2(x-1)).$$

   ii. Is the function $3f(2(x-1)) + 1$ a function of exponential type?

   iii. Sketch the graphs of $f(x), f(2x), f(2(x-1)), 3f(2(x-1))$ and $3f(2(x-1))+1$ in the same coordinate system and explain which graphical operation(s) (vertical shifting, vertical dilation, horizontal shifting, horizontal dilation) have been carried out.

   (b) In general, explain what happens when you apply the four construction tools of §2.5 (vertical shifting, vertical dilation, horizontal shifting, horizontal dilation) to the standard exponential model $y = A_o b^x$. For which of the four operations is the resulting function still a standard exponential model?

9. According to the *Merck Manual*, the relationship between the height h (in inches), weight w (in pounds) and surface area S (in square meters) for a human is approximated by the equation:

$$S = 0.0104 h^{0.425} w^{0.725} \text{ meter}^2$$

   Fix h to be your height, in inches. Let $S = S(w)$ be the resulting function computing your surface area as a function of your weight w. What is your surface area? How much weight must you gain for your surface area to increase by 5%? Decrease by 2%?

10. *Myoglobin* and *hemoglobin* are oxygen carrying molecules in the human body. The function

$$Y = M(p) = \frac{p}{1+p}$$

   calculates the fraction of myoglobin saturated with oxygen at a given pressure p torrs. For example, at a pressure of 1 torr, $M(1) = 0.5$, which means half of the myoglobin (i.e. 50%) is oxygen saturated. (Note: More precisely, you need to use something called the "partial pressure", but the distinction is not be important for this problem.) Likewise, the function

$$Y = H(p) = \frac{p^{2.8}}{26^{2.8} + p^{2.8}}$$

   calculates the fraction of hemoglobin saturated with oxygen at a given pressure p. Hemoglobin is found inside red blood cells, which flow from the lungs to the muscles through the bloodstream. Myoglobin is found in muscle cells.

   (a) Use a graphing device to sketch graphs of $M(p)$ and $H(p)$ in the same coordinate system; use the domain $0 \le p \le 100$ torrs.

   (b) If the pressure in the lungs is 100 torrs, what is level of oxygen saturation of the hemoglobin in the lungs?

   (c) The pressure in an active muscle is 20 torrs. What is the level of oxygen saturation of myoglobin AND of hemoglobin in an active muscle?

   (d) Define the efficiency of oxygen transport at a given pressure p to be $M(p) - H(p)$. What is the oxygen transport efficiency at 20 torrs? At 40 torrs? At 60 torrs? Interpret these transport efficiencies in terms of the graph you drew in part (a).

11. A colony of yeast cells is estimated to contain 10^6 cells at time $t = 0$. After collecting experimental data in the lab, you decide that the total population of cells at time t hours is given by the function

$$y = 10^6 e^{0.495105t}.$$

(a) How many cells are present after one hour?

(b) (True or False) The population of yeast cells will double every 1.4 hours.

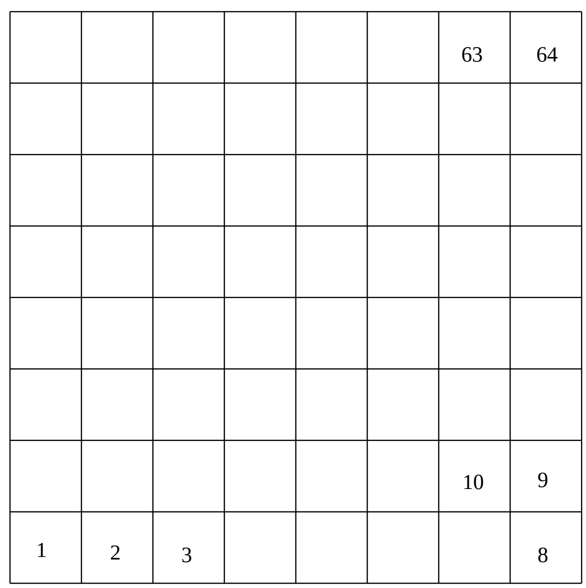
(c) Cherie, another member of your lab, looks at your notebook and says : ...that formula is wrong, my calculations predict the formula for the number of yeast cells is given by the function

$$y = 10^6 (2.042727)^{0.693147t}.$$

Should you be worried by Cherie's remark?

(d) Anja, a third member of your lab working with the same yeast cells, took these two measurements: 7.246×10^6 cells after 4 hours; 16.504×10^6 cells after 6 hours. Should you be worried by Anja's results? If Anja's measurements are correct, does your model over estimate or under estimate the number of yeast cells at time t?

12. After watching the Road Runner on TV, you decide to play a game. You have a chess board as pictured, with squares numbered 1 through 64. You also have a huge change jar with an unlimited number of dimes. On the first square you place one dime. On the second square you stack 2 dimes. Then you continue, always *doubling* the number from the previous square.



(a) How many dimes will you have stacked on the 10th square?

(b) How many dimes will you have stacked on the nth square?

(c) How many dimes will you have stacked on the 64th square?

(d) Assuming a dime is 1 mm thick, how high will this last pile be?

(e) The distance from the earth to the sun is approximately 150 million km. Relate the height of the last pile of dimes to this distance.