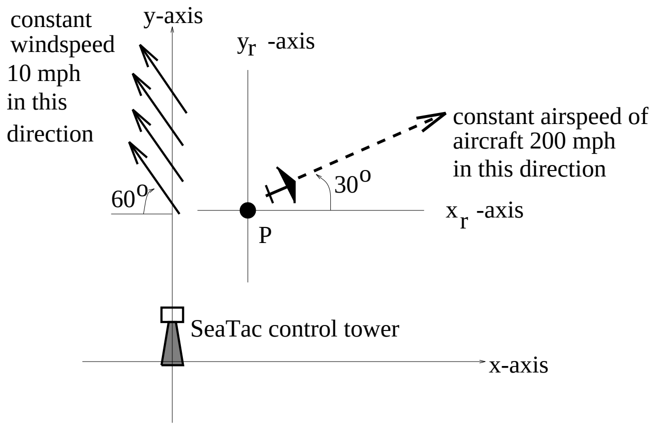
4.5 Combined Motion

1. Impose an xy-coordinate system with the control tower of SeaTac airport at the origin. At 9:34:12 am, an airplane is identified by radar at a location 20 miles North and 10 miles East of the control tower. The pilot radios they are flying at a constant altitude of 10,000 feet, with a constant airspeed of 200 mph in the direction pictured. The windspeed over the ground is 10 mph in the direction pictured.



   (a) Let $(x_c(t), y_c(t))$ denote the location of the initial identification position in the airmass at time t; t in seconds. This will be the origin of a relative $x_r y_r$-coordinate system. Using the wind information, find parametric equations for $(x_c(t), y_c(t))$.

   (b) Find the location $(x_r(t), y_r(t))$ of the plane in the relative $x_r y_r$-coordinate system.

   (c) Describe the location $(x(t), y(t))$ of the plane at time t seconds after identification.

   (d) Assuming no changes in the data given, where is the plane located at 9:35 am?

   (e) At what time will the aircraft be 35 miles from the SeaTac control tower?

   (f) What is the actual direction and speed of the airplane over the ground?

2. A vintage World War I biplane is spotted at a location 1 mile North and East of a bridge, flying at a level altitude heading 20^o counterclockwise from East with a constant airspeed of 48 mph. The wind is blowing at a constant speed of 65 mph with a direction angle of 205^o.

   (a) Describe the location $(x(t), y(t))$ of the plane at time t seconds after identification.

   (b) Where is the plane located 10 minutes after spotting? Is something funny here?

   (c) What is the direction and groundspeed of the airplane?

3. An airplane wishes to fly a course due North with a groundspeed of 120 mph. The wind is blowing 50 mph from the West to the East. What is the required airspeed and heading of the plane?

4. A bicyclist is moving at a constant speed of 24 mph along level ground. A pebble sticks in the tread of the rear tire, which has a radius of 15 inches.

   (a) Describe the parametric equations $(x(t), y(t))$ for the motion of the pebble relative to an observer on the ground.

   (b) Find the location of the pebble after one second and after one minute.

(c) During the first one second, how many times does the pebble touch the ground?

(d) What is the maximum possible height of the pebble above the ground? How often is this height achieved during the first second?

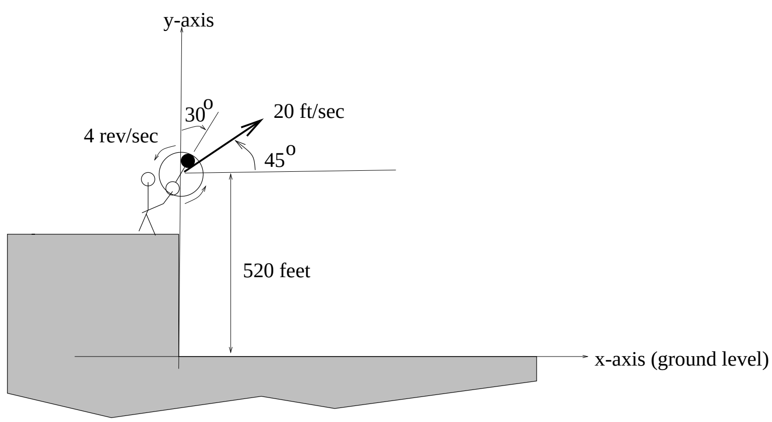
5. Return to Example 4.5.5 with the same data and focus on the white tip of the baton.

   (a) Find parametric equations for the motion of the white tip; the picture of the path followed by this tip is at the end of §4.5.4.

   (b) At the instant when the center of the baton is 20 feet above the ground, what is the location of the white tip? What is the orientation of the baton at this instant?

   (c) Is the white tip of the baton initially moving forward or backward ?

6. A person standing at the edge of a tall cliff is spinning a baton of length 2 feet so that the center of the shaft is 520 feet above the level ground below. One end of the baton is black and the other end is white. We assume the baton is spinning 4 rev/sec counterclockwise in the same plane as the trajectory, so that we have a two-dimensional problem. At the moment when the black tip is in the 1 o'clock position (30^o right of vertical), the baton is tossed at 20 feet/sec at an angle of 45^o above horizontal, as pictured below.



   (a) Find parametric equations for the motion of the black tip.

   (b) At the instant when the center of the baton reaches its maximum height, what is the location of the white tip? What is the orientation of the baton at this instant?

   (c) Give an estimate as to where the baton lands and justify your claim in some way.

7. Returning to Example 4.5.6, rework the problem given data:

   (a) The large platform has radius 30 feet rotating 4 RPM counterclockwise and the circular seat has radius 4 feet rotating $\frac{1}{2}$ rev/sec clockwise.

   (b) The large platform has radius 20 feet rotating 15 RPM clockwise and the circular seat has radius 10 feet rotating 2 rev/sec counterclockwise.

8. Return to Example 4.5.2. Determine the angle Buzz should have fired the puck to insure it would go into the goal.

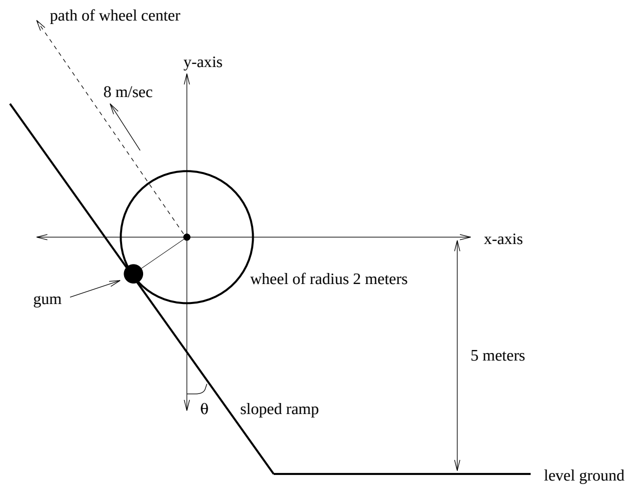
9. A wheel of radius 2 meters is rolling up an inclined ramp at a rate of 8 meters/second, as pictured. We impose a coordinate system so that the origin is the initial location of the center of the wheel. Assume the indicated angle $\theta = \frac{\pi}{5}$ radians.

(a) Find parametric equations for the motion of the center of the wheel at time t seconds.

(b) When is the center of the wheel 23 meters above the level ground?

(c) A blob of gum is stuck to the spot where the wheel initially touches the sloped ramp; a radial line from the initial wheel center to the gum is perpendicular to the sloped ramp. Find the location of the gum on the wheel.

(d) Find parametric equations for the motion of the gum blob at time t seconds.



10. Return to Example 4.5.2.

    (a) In the same coordinate system, plot these three linear graphs: the line of motion for the life preserver; the line of motion of the sailboat; the line connected the initial fix and Richardson.

    (b) Determine where the sailboat reaches the shore of Lopez Island.

    (c) Determine the speed and direction of the sailboat relative to the initial fix.

11. Return to Example 4.5.2. Assume Ken is located at the position of the initial fix. In what direction should Ken point his sailboat in order to sail directly to the dock at Richardson?

12. Buzz Gordon, world famous hockey player, is skating in a straight line at a constant speed of 30 ft/sec. He crosses the rink centerline (the x-axis pictured below) 25 feet to the right of a practice target. The instant Buzz reaches the location $P = (20, 10)$ in the pictured coordinate system (distance units are feet), he fires the puck 50 ft/sec at an angle of 60^o below horizontal. Find parametric equations for the motion of the puck. Determine where the puck crosses the x-axis and y-axis, concluding Buzz misses the target.