4.4 Projectile Motion

1. A ball rolls off the edge of a cliff with a height of 1234 feet above the level ground below. As the ball leaves the top of the cliff it is moving horizontally with a speed of 18mph.

   (a) Find parametric equations for the motion of the ball.

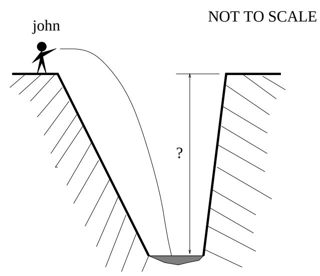
   (b) What is the position of the ball 7 seconds after leaving the edge of the cliff?

   (c) When and where does the ball hit the ground?

   (d) A person standing 340 feet in front of the cliff observes the event. Does the ball land in front of or behind the observer?

   (e) Find a quadratic function $y = f(x)$ a portion of whose graph is the trajectory.

2. John is standing near the edge of a deep canyon with a river in the bottom. He throws a rock into the canyon so that its initial velocity v is entirely in the horizontal direction. John counts that it takes 5 seconds for the rock to splash into the water. Estimate the depth of the canyon.

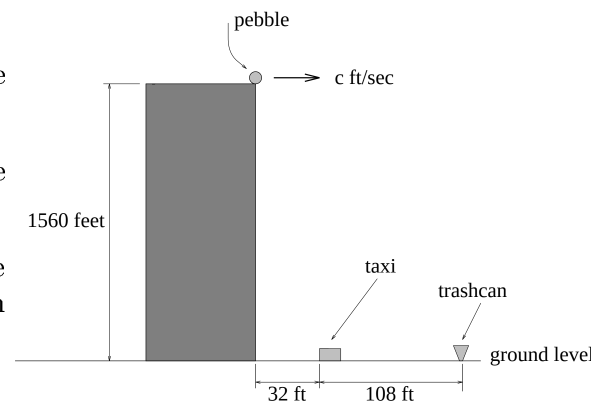


3. While standing atop a 1560 feet high office building you notice a round pebble on the edge of the roof. Suppose you gently push the pebble over the edge. Assume that the pebble has a constant horizontal velocity c feet/sec. Furthermore, imagine the pictured situation below:

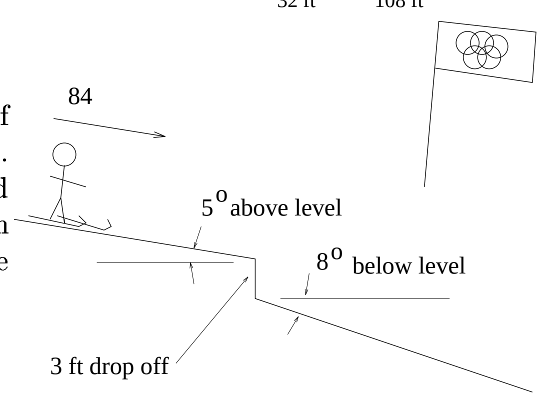
   (a) If c=0.1 ft/sec, where does the pebble land on the ground?

   (b) Find the largest value for c so the pebble lands on the building side of the taxi.

   (c) Find the required value for c so that the pebble lands in the middle of the trash can.



4. A downhill ski racer has a constant velocity of 84 miles per hour on a downward sloping ramp. The ramp slopes down 5^o below horizontal and the ramp drops 3 feet to another sloped portion of the course. This lower portion of the course has a constant slope of 8^o below horizontal:



   (a) Impose a coordinate system and find parametric equations for the airborne skier.

   (b) Where and when is the skier 1 ft vertically above the lower portion of the course?

   (c) Where and when does the skier land on the lower part of the course?

(d) Find the maximum height of the skier above the course, when this happens and the location of the skier at this instant.

5. Two pine cones release at the same instant from an old growth Douglass Fir. At the instant the cones release, a side wind to the right (away from the tree) causes them to move with a horizontal velocity of 25 mph. Assume that the ground slopes up from the base of the tree at a constant rate of 1 vertical foot for each 10 horizontal feet; see picture below. Impose a coordinate system as pictured and answer these questions:

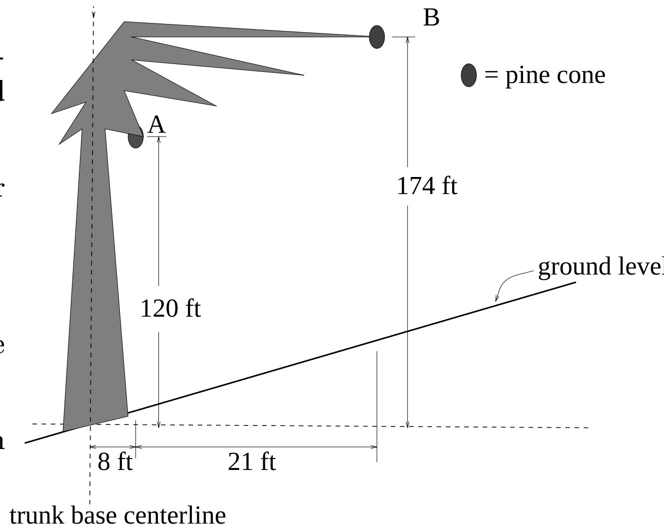
(a) Give parametric equations for the trajectory of each of the pine cones A and B.

(b) Describe the location of each cone after $\frac{3}{2}$ seconds have elapsed.

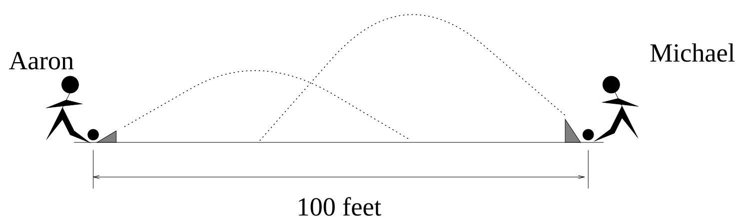
(c) When does each cone hit the ground?

(d) When is each cone 10 feet above the ground?

(e) Describe the location of cone B when cone A hits the ground.



6.* Michael is located 100 feet to the right of Aaron as pictured below. They are both about to kick a ball.



(a) Introduce a common coordinate system with Aaron as the origin.

(b) Aaron kicks a ball with an initial speed of 40 ft/sec in a direction 40^o above horizontal. What is the parametrized trajectory of this ball?

(c) Michael kicks a ball toward Aaron with an initial speed of 60 ft/sec in a direction 70^o above horizontal. What is the parametrized trajectory of this ball?

(d) Find quadratic functions $y = f(x)$ and $y = g(x)$ which describe the trajectories of each ball.

(e) Using this information, find where the two trajectories intersect (as in the picture).

(f) Describe when each ball should be kicked so that they collide in midair.

7. A beach ball of diameter 24 inches is kicked from the edge of 23 ft high bluff above a level beach. The velocity vector for the ball is 30 feet/sec in a direction 51^o above horizontal.

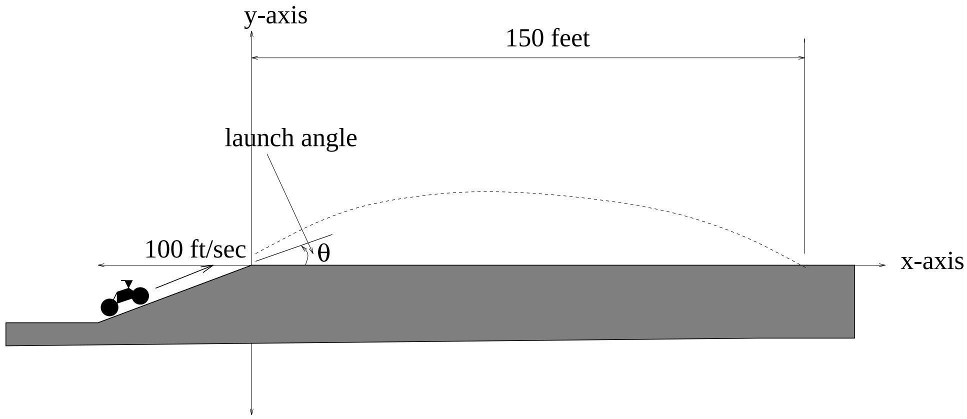
(a) Sketch a picture of the situation.

(b) Describe the trajectory of the center of the ball. You need to be careful here, since the radius of the ball must be taken into account. Namely, the center of the ball is actually 24 feet above the level beach when kicked.

(c) When and where does the ball hit the ground? (Keep in mind that the ball hits the beach precisely when the center is 1 ft above the ground.)

8. Redo Example 4.4.12 with the following change: The cyclist is traveling 20 mph and the maximum height is 12 feet above the landing zone.

9. A stunt cyclist needs to make a calculation for an upcoming cycle jump. The cyclist is traveling at a speed of 100 ft/sec toward an inclined ramp which is followed by a level landing zone (no drop off as in the previous example). Assume the rider maintains speed up the ramp, which has angle θ above horizontal. The cyclist needs to land 150 feet in front of the launch point. Find the desired angle of launch. (Hint: There will be two answers; use the ideas in Example 4.4.12. The double angle formula is also useful: $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$.)



10. A ground launched cannon fires a projectile with a constant speed of 1,000 feet/sec.

    (a) Determine the required angle(s) of launch if the target has the same elevation as the cannon and is located 5.8 miles away.

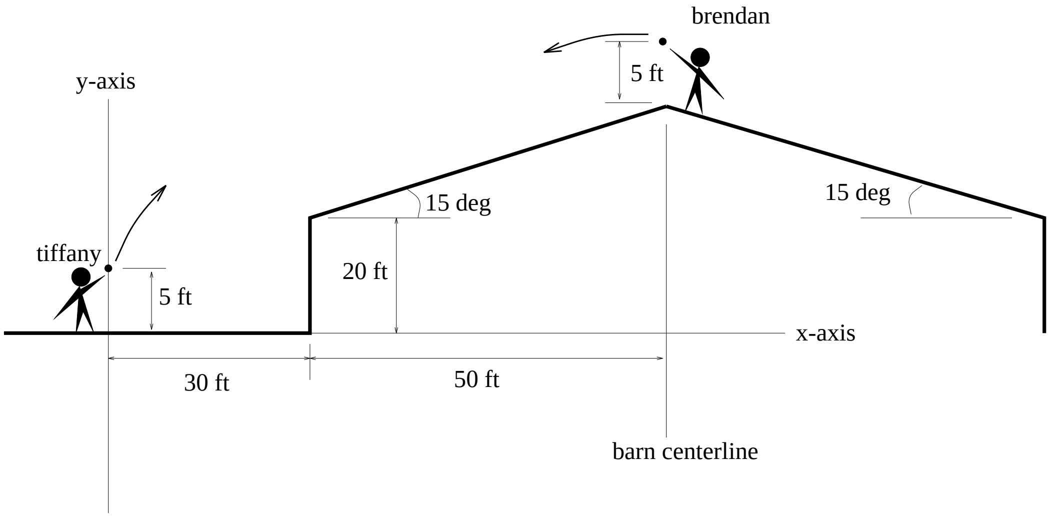
    (b) Describe the trajectory of the projectile for each launch angle in a., also determining the maximum height and elapsed time to impact.

    (c)* A target is 1 mile horizontally in front of the cannon AND 1000 feet above the elevation of the canon. Determine the required angle(s) of launch to strike the target.

11. Describe the portion of the parabola traced out by the trajectory in Example 4.4.11.

12. On the moon, the height of an object t seconds after being dropped from an initial height y_o feet above a reference level is given by $y = -2.62t^2 + y_o$; compare to the formula for the earth in (4.4.2). Rework Example (4.4.11), assuming you are on the moon.

13.* Tiffany and Brendan spent last summer roofing the barn on Tiffany's family farm. Brendan is a Jolt Cola drinker, so Tiffany was frequently throwing him refills. In turn, Brendan tossed back the empties. Suppose Tiffany tosses a can at an angle of 60^o above horizontal with a speed of 50 ft/sec. At the same instant, Brendan tosses an empty can in the direction indicated at an angle 3^o BELOW horizontal with a speed of 60 ft/sec.



(a) Find parametric equations $P(t) = (x_1(t), y_1(t))$ for the motion of Tiffany's tossed can.

(b) Where is Tiffany's tossed can located when it reaches its highest point above the ground?

(c) Find a function $y = f(x)$ whose graph is the path of Tiffany's tossed can.

(d) Where and when does Tiffany's tossed can hit the roof?

(e) Find parametric equations $Q(t) = (x_2(t), y_2(t))$ for the motion of Brendan's tossed can.

(f) Where is Brendan's tossed can located the instant it is no longer above the barn; i.e. when it has x coordinate 30?

(g) Find a function $y = g(x)$ whose graph is the path of Brendan's tossed can.

(h) If Tiffany doesn't move, should she be able to catch the can? How much time will she have to react?

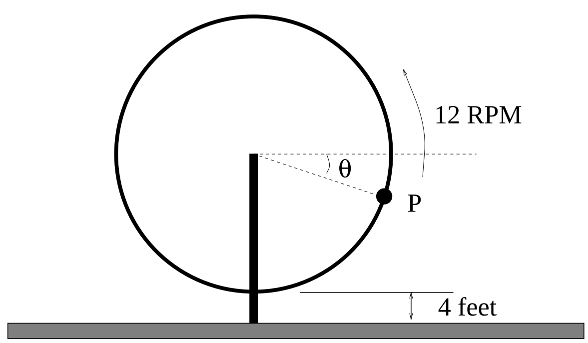
(i) Where are the two cans located the instant they have the same height above the ground?

(j) Where do the two trajectories cross one another?

(k) Use a graphing device to find the minimum distance between the two airborne cans.

14.* Return to the "magical marble toss" in Exercise 2.3.18. Find the angle and the speed of launch of the marble.

15. John has been hired to design an exciting carnival ride. Tiff, the carnival owner, has decided to create the worlds greatest ferris wheel. Tiff isn't into math; she simply has a vision and has told John these constraints on her dream: (i) the wheel should rotate counterclockwise with an angular speed of 12 RPM; (ii) the linear speed of a rider should be 200 mph; (iii) the lowest point on the ride should be 4 feet above the level ground. Recall, we worked on this problem in Exercises 3.2.14, 3.3.16 and 4.3.4. In Exercise 4.3.4, we resolved the velocity vector for Tiff's motion the instant she becomes a human missle.



(a) Find the parametric equations for Tiff's motion once she becomes a human missile.

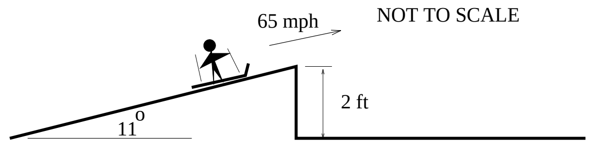
(b) Find where Tiff is located 6 seconds after she becomes a missile.

(c) Find when Tiff is 471 feet above the ground after she becomes a missile.

(d) Find where and when Tiff has maximum height above the ground.

(e) Find where and when Tiff craters (i.e. lands).

16. Nora is skiing out of control at 65 miles per hour up a ramp having an angle of 11^o above horizontal. The end of the ramp drops 2 feet to a level portion of the course.



(a) Impose a coordinate system.

(b) Resolve the velocity vector of Nora as she goes off the ramp.

(c) Find the coordinate function $x(t)$ for Nora, where $t = 0$ is the instant she is airborne.

(d) Find the coordinate function $y(t)$ for Nora, where $t = 0$ is the instant she is airborne.

(e) Use $y(t)$ to find where and when Nora lands on the level portion of the course.

(f) Use $y(t)$ to find Nora's maximum height above the level part of the course and when this happens.

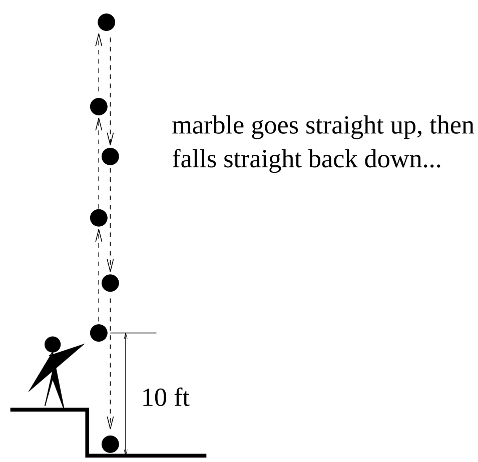
(g) Describe the trajectory of Nora; i.e. her path as she flies off the jump.

(h) Where and when is Nora 1 inch above the level portion of the course?

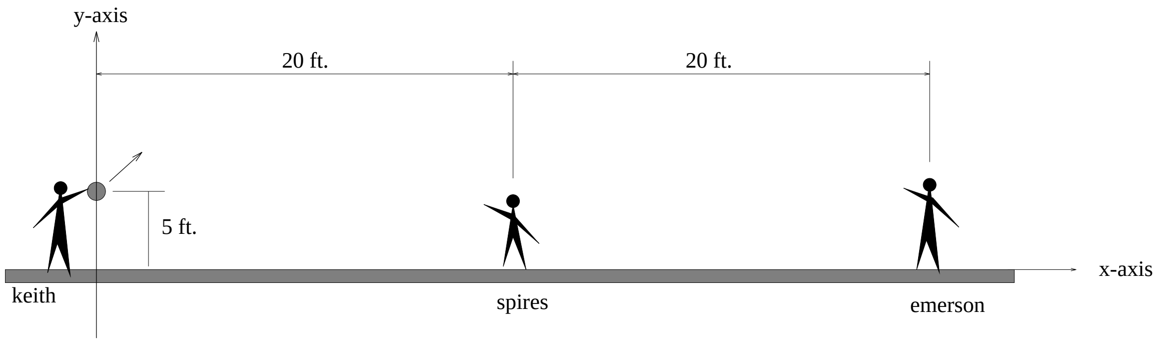
(i) Lee, who is 6 feet tall, is adjusting the straps on his snowboard while he stands 50 feet to the right of the ramp. Does Nora fly over Lee or land in front of him?

(j) Adri-N flys off the ramp after Nora with a speed of s mph. Find the smallest speed s so that Adri-N flies over Lee.

17. Adri-N is standing on the edge of a cliff and throws a marble straight up into the air at 75 ft/sec; the marble is released when it is $10\,ft$ above ground. It goes straight up, then falls back down to the ground. Find the TOTAL distance the marble travels between the time it is released and the time it hits the ground.



18. Three basketball players are in the locations below; impose a coordinate system as pictured. Keith throws the ball toward Spires. Let $P(t) = (x(t), y(t))$ be the location of the ball t seconds after Keith passes it; distance units will be feet. Assume $x(t) = 22t$ and $y(t) = -16t^2 + 20t + 5$. We will make life simple and view motion of the basketball as the motion of its center.

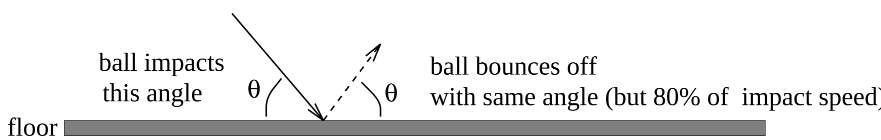


(a) Compute the initial velocity vector for Keith's pass.

(b) Keith's pass to Spires is a wild one; why?

(c) The ball soars over Spires. Determine the location of the ball when it hits the floor.

(d) Assume the ball bounces off the floor with the same angle $\theta = 50.64^o$ as it impacts the floor; see picture below. Also assume the impact speed of the ball is 34.68 ft/sec and the ball bounces with 80% of the impact speed.



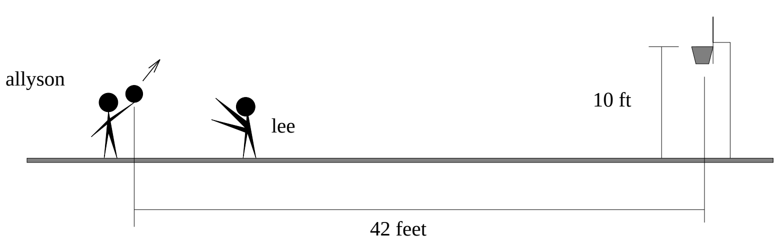
What are the parametric equations for the motion of the ball bouncing off the floor?

(e) Should Emerson be able to handle the ball? (Justify your answer.)

(f) Find the multipart function whose graph models the path of Keith's pass and sketch the graph.

19. Once again, Allyson is playing Lee in a game of one on one. Lee is leading 16-14. Just before time runs out, Allyson makes a three pointer and Lee limps away in defeat (again). Take the origin of the coordinate system to be on the ground beneath the ball. The equation of the path of the ball is given by

$$y = -0.0729x^2 + 3.16x + 6.$$



If Allyson shoots the ball at time $t = 0$, when does the ball go through the hoop?

20. Consider the problem of a baseball pitcher who throws a ball at 145 feet/second from a height of 6 feet. Suppose the ball is thrown at an angle (above or below the horizontal) of θ degrees. How do we calculate the angle θ so that the ball will cross home plate at a certain height?

(a) What is the initial horizontal velocity of the ball, expressed in terms of θ?

(b) What is the initial vertical velocity of the ball, expressed in terms of θ?

(c) Write parametric equations for the position of the ball at time t. Your answer will involve the constant θ.

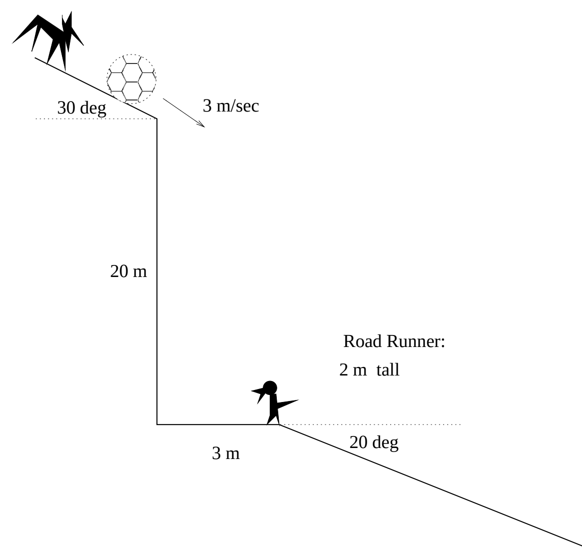
(d) Let's call t_p the time at which the ball crosses home plate. In baseball, home plate is 60.5 feet from the pitcher. Thus t_p is the time at which the ball has traveled horizontally a distance of 60.5 feet. Find t_p, in terms of θ.

(e) At the time t_p, what is the height y_p of the ball above the ground? Again, your answer will involve the constant θ and trigonometric functions of θ.

(f) Rewrite your answer for the value of y_p in terms of $\tan(\theta)$, using the identity $\tan^2(\theta)+1 = 1/\cos^2(\theta)$.

(g) Suppose the pitcher wants the ball to cross home plate at a height of 2 feet above the ground. For this to occur, what must θ be? You should get two answers; why?

21. Wile E. Coyote has an ACME ordered boulder which he is rolling off a 20 m high cliff as shown, trying to squash the Road Runner.



(a) Impose a coordinate system in the picture at right.

(b) Find parametric equations for the coordinates of the boulder with $t = 0$ corresponding to the instant it rolls off the cliff.

(c) Will the boulder hit the Road Runner?

(d) Find the equation $y = f(x)$ of the sloping ground beside the Road Runner.

(e) What are the coordinates of the point where the boulder finally hits the ground?