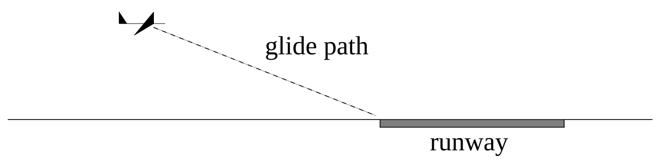
## 4.3 A Crash Course in Mechanics

1. Suppose a car is driving up a mountain pass at a constant speed of 55 mph. Assume the road is inclined at a steady angle of $8^o$ above horizontal.

   (a) Resolve the velocity vector $v$ for the car.

   (b) If the mountain pass is located at 5800 feet and the car has just passed a sign "Elevation 4500 feet", how long before you reach the pass?

2. An airplane is on its glide path for landing at the airport as pictured below.

   According to the altimeter, the altitude is decreasing at the constant rate of 200 feet/min and the control tower reports your horizontal ground speed is 81 mph; assume no wind.

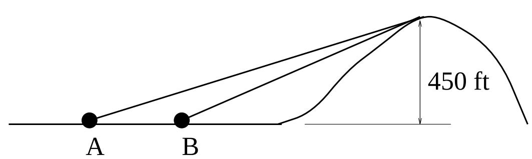
   (a) Describe the velocity vector for the plane.

   (b) What is the airspeed of the plane?

   (c) If the airfield is at an elevation of 1250 feet and the plane altimeter reads 1800 feet, how much time elapses before touchdown?

   (d) If the airplane is 2 miles in front of the runway, how much time elapses before touchdown?

3. Two ramps lead from ground level to the top of a hill, one having an incline of $17^0$ and the other having an incline of $22^o$.

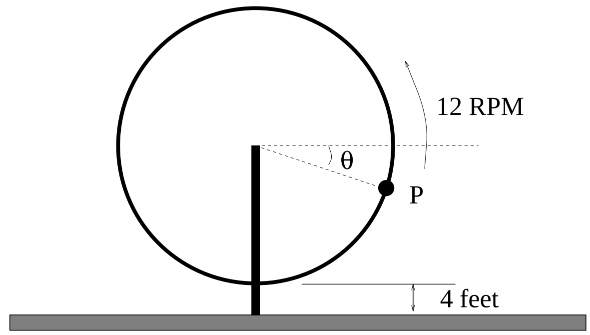
   (a) What is the length of each ramp?

   (b) Suppose runners at points $A$ and $B$ in the picture begin running up the ramps at a constant speed. Assume runner $A$ (the runner starting at $A$) runs at a constant speed of 8 mph up the ramp. Resolve the velocity $v^A$ of runner $A$.

   (c) How long does it take runner $A$ to reach the top of the hill?

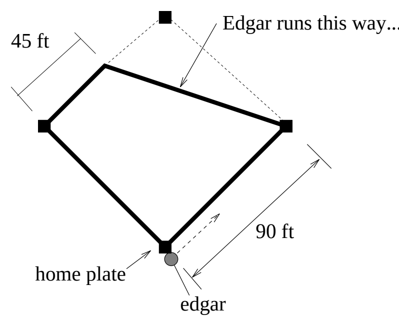
   (d) What is the required velocity $v^B$ for runner $B$ (the runner starting at $B$) to reach the top of the hill at the same time as runner A?

4. John has been hired to design an exciting carnival ride. Tiff, the carnival owner, has decided to create the worlds greatest ferris wheel. Tiff isn't into math; she simply has a vision and has told John these constraints on her dream: (i) the wheel should rotate counterclockwise with an angular speed of 12 RPM; (ii) the linear speed of a rider should be 200 mph; (iii) the lowest point on the ride should be 4 feet above the level ground. In Exercise 3.3.16, we determined the location and tangential direction for Tiff's motion the instant she becomes a human missile. Resolve the vector for Tiff's motion the instant she launches.




5. Return to Exercise 2.2.9. Find parametric equations for Edgar's location at time $t$ (each coordinate function will be a multipart function).

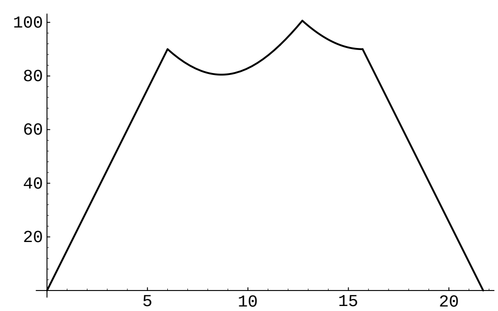
6.* This problem is a variation on the previous problem. Suppose Edgar hits a long ball, but is confused and runs a speed of 15 ft/sec along the path pictured.




(a) Find parametric equations for Edgar.

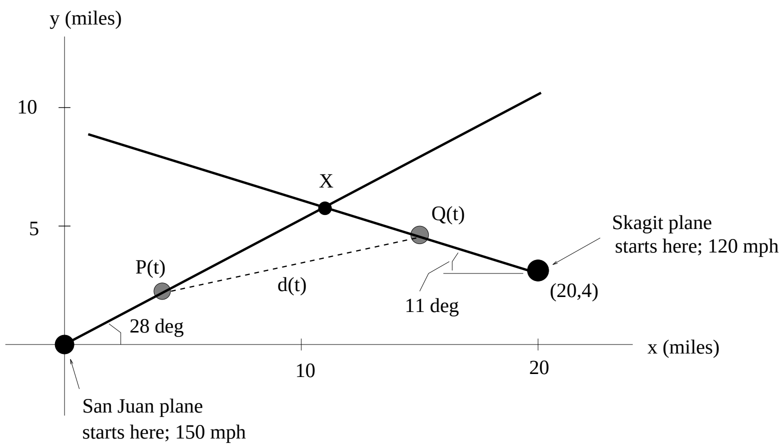
(b) Find a formula for the function $d(t)$ which computes the distance from home plate to Edgar at time $t$ seconds.

(c) Here is a plot of the graph of $d(t)$; use your graphing device to verify this. During the time Edgar is between first and third base, determine where and when he is closest to home plate. Interpret your answer using the graph of $d(t)$.




7. (a) If $v_x = -10$ ft/sec and $v_y = 35$ ft/sec, find $|v|$ and the direction of $v$; sketch a picture.
   (b) If $v_x = -34$ ft/sec and $v_y = -12$ ft/sec, find $|v|$ and the direction of $v$; sketch a picture.
   (c) If $v_x = -15$ ft/sec and $|v| = 35$ ft/sec, find the possible $v_y$ and directions of $v$.
   (d) If $v_y = 8$ ft/sec and $|v| = 113$ ft/sec, find the possible $v_x$ and directions of $v$.

8.* Two airplanes are initially spotted as pictured below. The San Juan flight is initially spotted at the origin of the imposed coordinate system and flies along pictured path 150 mph. The Skagit flight starts at the location (20,4) and flies 120 mph along pictured path. The units in the diagram will be miles:




(a) Find functions $y = f(x)$ and $y = g(x)$ whose graphs are the flight paths of the San Juan and Skagit flights, respectively.

(b) Find the point $X$ where the two flight paths cross. When will each plane reach $X$? Do the planes collide?

(c) Find the parametric equations $P(t) = (x_1(t), y_1(t))$ for the San Juan flight; here, $t$ will be in hours.

(d) Find the parametric equations $Q(t) = (x_2(t), y_2(t))$ for the Skagit flight; here, $t$ will be in hours.

(e) Where is each plane located after 3 minutes?

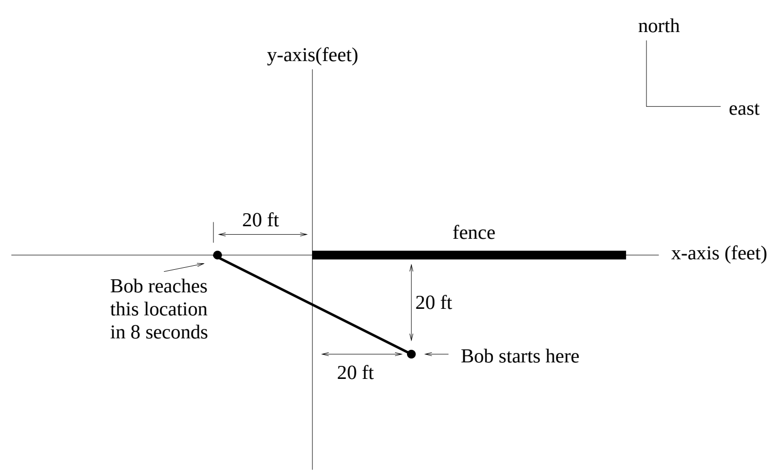
(f) Using the distance formula, find a function $d(t)$, in the time variable, which computes the distance between the two planes $t$ hours after they are spotted.

(g) Use the function $d(t)$ in the previous part to find the distance between the two planes after 3 minutes.

(h) Find the minimum distance between the two planes and determine when this happens. (Hint: Minimize the function $(d(t))^2$ first.)

(i) Where are the two planes located (in the picture) when they are closest to one another?

9. A fence with one end located at the origin in the $xy$-plane extends to the east as shown. (Here, $x$ and $y$ coordinates are the distances east and north of the origin.) Bob runs from the location 20 feet east and 20 feet south of the origin at constant speed, reaching the location 20 feet directly west of the origin 8 seconds later.



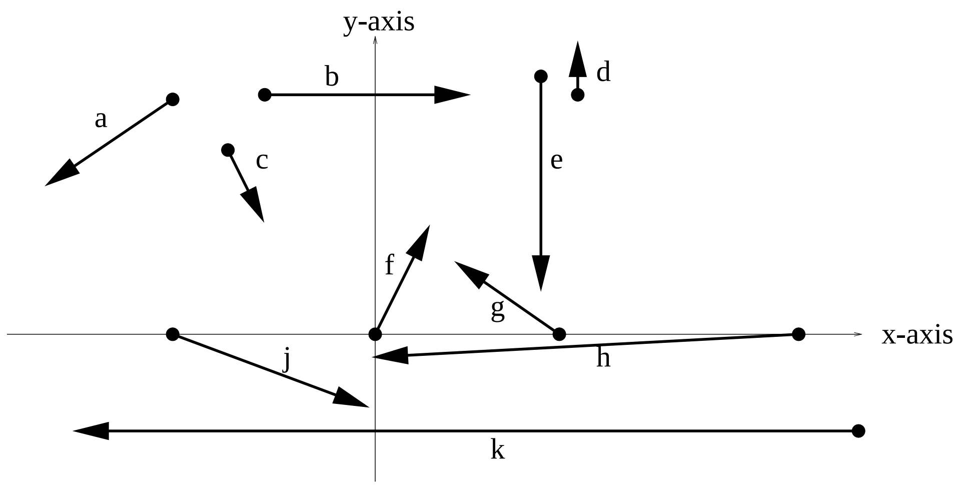

(a) Find the coordinates of Bob after $t$ seconds for $0 \le t \le 8$; i.e. find parametric equations for Bob's motion.

(b) Find the distance from Bob to the fence after $t$ seconds. (Note: This is a multipart function.)

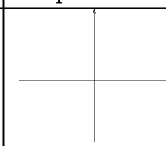
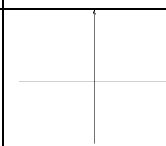
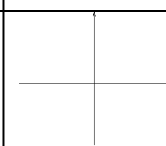
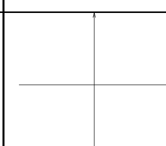
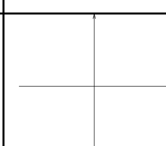
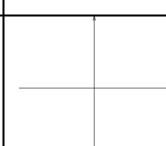
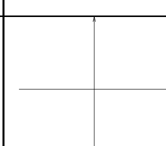
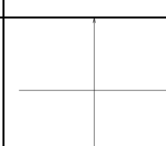
(c) Find the time(s) and location(s) where Bob is exactly 15 feet away from the fence?

10. The picture below shows several different vectors $v = a, b, c, d, e, f, g, h, j, k$. For each vector $v$, answer these questions:

- Is $v_x$ positive, negative or zero?
- Is $v_y$ positive, negative or zero?
- Does the direction angle $\theta$ for the vector satisfy the condition: $0 \leq \theta \leq \pi/2$?
- Does the direction angle $\theta$ for the vector satisfy the condition: $\pi/2 \leq \theta \leq \pi$?
- Does the direction angle $\theta$ for the vector satisfy the condition: $-\pi \leq \theta \leq 0$?
- Which vector has $|v_x|$ the largest? Which vector has $|v_x|$ the smallest?
- Which vector has $|v_y|$ the largest? Which vector has $|v_y|$ the smallest?



11. Complete the following table:

| equation of path $y = f(x)$ | $x(t)$ | $y(t)$ | $v_x$ | $v_y$ | $\|v\|$ | direction angle | starting point | picture |
|---|---|---|---|---|---|---|---|---|
| | | | | | 8 | $\frac{5\pi}{4}$ | $(2, 3)$ |  |
| | | | -4 | 3 | | | $(1, 2)$ |  |
| | $5t + 3$ | -3t+7 | | | | | |  |
| $y = 3x + 2$ | $-2t - 1$ | | | | | | |  |
| $y = 3x + 2$ | $2t - 1$ | | | | | | |  |
| | | | | | 2 | 0 | (1,-1) |  |
| | | | | | 2 | $\pi$ | (1,-1) |  |
| | | 3t+2 | | | | $2rad$ | (0,2) |  |