4.2 Linear Motion

1. Return to Example 4.2.1, but assume the bug begins at Q and walks toward P. Again assume the bug walks at a constant speed, requiring 5 seconds to traverse the distance.

   (a) Describe the motion of the bug pictorially and via parametric equations.

   (b) Find the speed and direction of the bug along the line connecting Q to P? How about the horizontal and vertical speeds?

   (c) How far is the bug from the origin after 2 seconds?

2. The fact stated in (4.2.6) tells us that if a bug travels at a constant speed along a straight line connecting P and Q, then the speeds in the horizontal and vertical directions will both be constant. This is a *very special* property of linear motion and this exercise shows we must be careful. Suppose a bug begins at the location $P = (1, 0)$ in the xy plane and travels counterclockwise along the unit circle to the point $Q = (0, 1)$. Assume the bug moves at a constant speed and takes 3 seconds to travel this path.

   (a) Sketch a picture of the situation being studied.

   (b) Compute the bugs constant speed s from P to Q by dividing distance traveled by time elapsed.

   (c) Prepare a convincing argument that neither the horizontal nor vertical speed of the bug is constant.

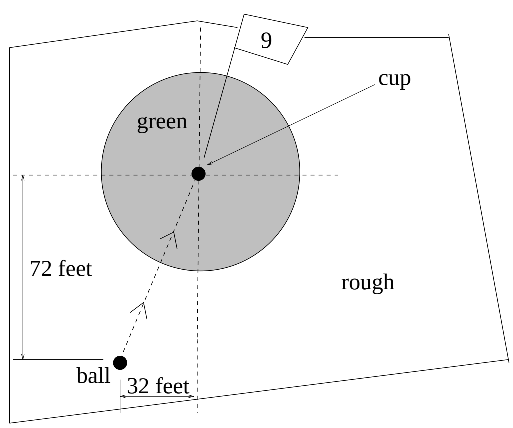
3. The cup on the 9^{th} hole of a golf course is located dead center in the middle of a circular green which is 70 feet in diameter. Your ball is located as in the picture below. Assume that you putt the ball at a constant speed in a straight line into the cup. Assume it takes 6 seconds to land in the cup.

   (a) How far does the ball travel?

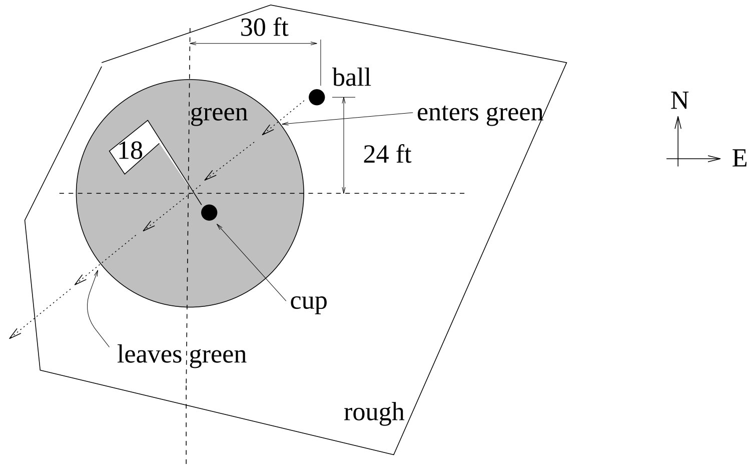
   (b) Find parametric equations for the motion of the ball.

   (c) How far is the ball from the cup after 3.5 seconds?

   (d) Where and when does the ball cross from the rough onto the green?



4. The cup on the 18^{th} hole of a golf course is located 4 feet south and 4 feet east of the center of a circular green which is 54 feet in DIAMETER. Your ball is located as in the picture below. Assume that you mistakenly putt the ball which travels at a constant speed in a straight line toward the center of the green. It takes 7 seconds to reach the center of the green. Use distance units of FEET and time units of SECONDS in this problem.



(a) Find parametric equations for the motion of the ball.

(b) How far is the ball from the cup after 5 seconds?

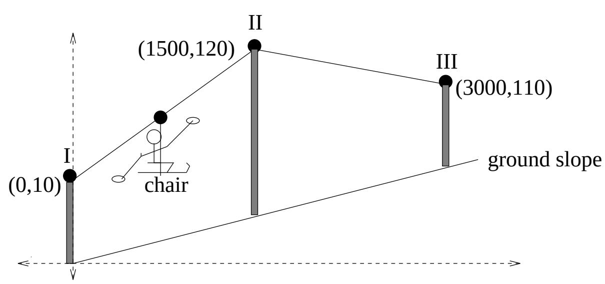
(c) Where and when does the ball cross into the green and out of the green?

(d) Find when and where the ball is closest to the cup. (Hint: Construct a line thru the cup perpendicular to the path of the ball, then find where these two lines intersect.)

(e) Find a function of t that computes the distance from the ball to the cup at any time t.

(f) When and where is the ball 8 feet from the cup?

5. A ski lift uses a cable running between a series of three towers. In the picture below, the leftmost tower is the beginning and the rightmost tower is the end. We will focus on describing the motion of your skis. When you first sit in the chair, the skis just touch the ground at the location (0,0) in the picture:



When you exit the lift, again your skis just touch the ground. The cable travels at a constant speed of 10 feet/second. Assume the cables do not sag and the hill slopes up at a constant rate of $m = \frac{1}{30}$.

(a) Describe parametric equations for the motion of your skis between towers I and II.

(b) Describe parametric equations for the motion of your skis between towers II and III.

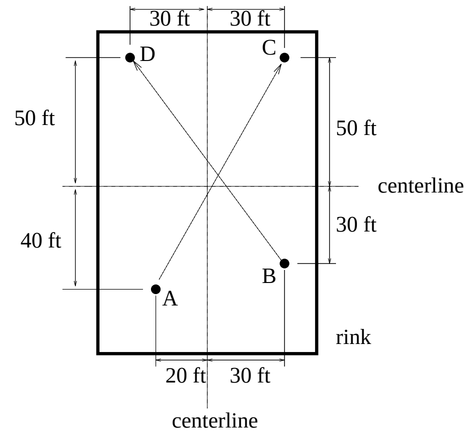
(c) What is the total distance your skis travel from start to finish?

(d) Describe the precise location of your skis after 1 minute. 4 minutes. 5 minutes.

(e) Where and when are your skis located 50 feet above the ground?

(f) When do you arrive at the finish?

6. Two practicing hockey players located at positions A and B in the picture. The player at position A strikes a puck, which we will assume moves at a constant speed along the line connecting A and C; assume it takes 2 seconds for the puck to go from A to C. At the same instant, the player at position B strikes a puck, which we will assume moves at a constant speed along the line connecting B and D; also assume it takes 2 seconds for the puck to go from B to D.



(a) Describe parametric equations for the motion of each puck.

(b) Describe the location of each puck after 1.4 seconds.

(c) Compute the distance between the two pucks after 1.4 seconds.

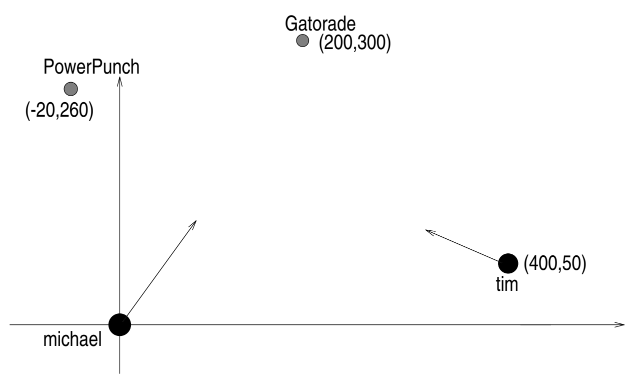
(d) Write down a function $d = d(t)$ which computes the distance between the two pucks at time t

(e) When is the distance between the two pucks 40 feet?

(f) When is the distance between the two pucks 5 feet?

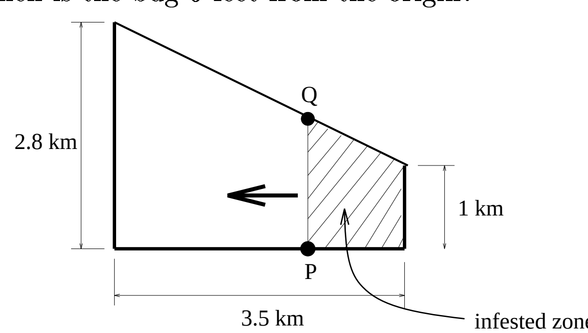
(g) Where do the paths of the two pucks cross?

7. After a vigorous soccer match, Tim and Michael decide to have a glass of their favorite refreshment. They each run in a straight line along the indicated paths at a speed of 10 ft/sec. Parametrize the motion of Tim and Michael individually. Find when and where Tim and Michael are closest to one another; also compute this minimum distance. (Hint: Use the strategy developed in Exercise 2.3.25. Begin by inserting the parametric equations for each person into the distance formula, leading to a function $d(t)$. Minimize this function.)



8. Return to Example 4.2.5. How far is the bug from the origin at time $t = 1$? Where and when will the bug be closest to the point (3,4)? When is the bug 6 feet from the origin?

9.* A Rocky Mountain Pine Beetle infestation is moving across Angela's eastern Washington ranch. The beetles move from right to left across the property, as pictured. Assume that the point P is moving at a speed of 100 meters/day to the left.



(a) When is the entire property trashed by the beetles?

(b) What is the speed of the infestation along the top boundary?

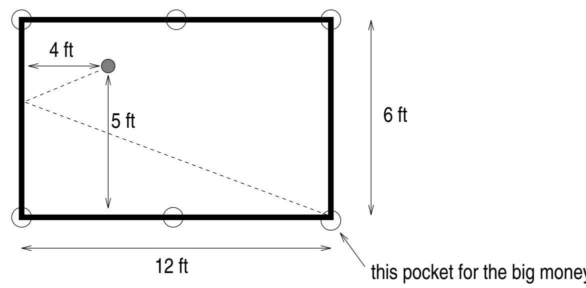
(c) Find a function which computes the area of the infested region as a function of time t.

(d) When will the infested region have area between 1 km^2 and 2 km^2?

(e) Find a function which computes the infested region perimeter as a function of time t.

(f) When does the infested zone have a perimeter of 10 km?

10. Recall the Billard problem in §3.3.1. Assume you strike the ball and it travels a constant speed of 20 in/sec along the dotted path.



(a) Find parametric equations for the motion of the ball. (Hint: Both the coordinate functions will be multipart functions.)

(b) Where is the ball located after 2 seconds?

(c) Where is the ball located after 5 seconds?

(d) When does the ball strike the cushion?

(e) When does the ball land in the pocket?

(f) When is the ball 2 feet from the pocket?