4.1 Parametric Equations

1. Return to Example 4.1.1. What are the coordinates of the bug in the six "snapshots" in the solution? When will the bug first cross into the second quadrant? When will the bug first have x-coordinate=-0.2?.

2. Sketch the curve represented by the parametric equations. Choose four specific t values and indicate the corresponding points on the curve. As t moves from left to right in the domain, indicate how the corresponding points on the curve are moving.

   (a) $x(t) = 1, y(t) = 4 - 2t, 0 \leq t \leq 5$.

   (b) $x(t) = t - 1, y(t) = 4, 0 \leq t \leq 5$.

   (c) $x(t) = t - 1, y(t) = 4 - 2t, 0 \leq t \leq 5$.

   (d) $x(t) = 4 - t, y(t) = 2t - 6, 0 \leq t \leq 5$.

   (e) $x(t) = -\cos(t), y(t) = \sin(t), 0 \leq t \leq 2\pi$.

   (f) $x(t) = \sin(t), y(t) = \cos(t), 0 \leq t \leq 2\pi$.

3. A charged particle is moving in the xy-plane. The distance units will be feet. The location of the particle at time t is given by the parametric equations: $x(t) = 5t + 1, y(t) = t^2 - 2t + 3$.

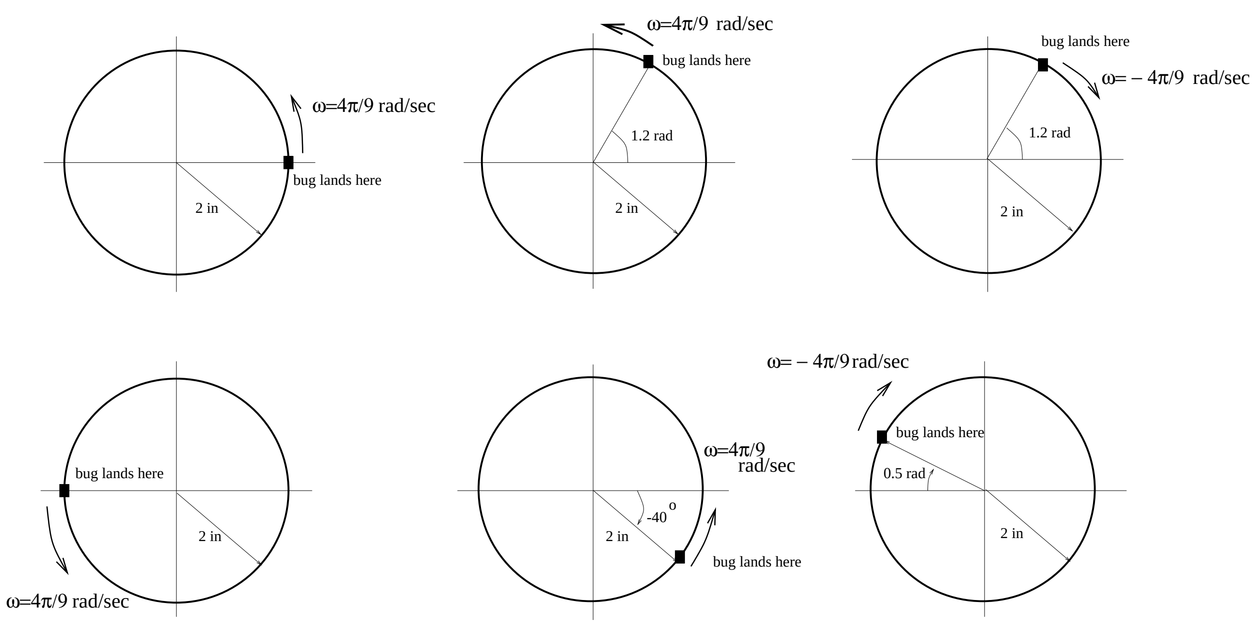
   (a) Find a function $y = f(x)$ whose graph coincides with the path of the particle.

   (b) Sketch a picture of the graph.

   (c) When is the particle closest to the x-axis? What is the particle location at this instant?

   (d) When is the particle 2.5 feet from the x-axis?

4. In the pictures below, a bug has landed on the rim of a jelly jar and is moving around the rim. The location where the bug initially lands is described and its angular speed is given. Impose a coordinate system with the origin at the center of the circle of motion. In each of the cases, find parametric equations describing the location of the bug at time t seconds.



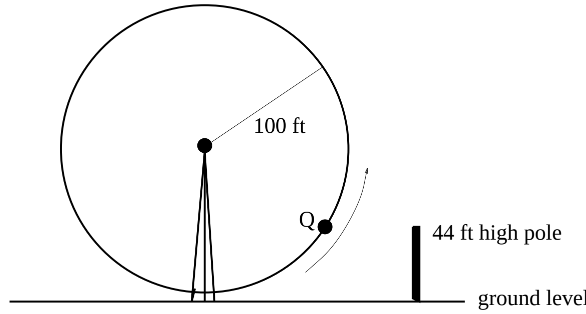
5. A rider hops on a merry-go-round at the compass point W on the platform. Assume the radius of the ride is 30 feet and it is rotating $\omega = \frac{\pi}{50}$ radians/second.

(a) Sketch a picture of the situation and impose a coordinate system.

(b) Find the parametric equations for the rider location at time t.

(c) After 38 seconds, where is the rider located?

6. A ferris wheel of radius 100 feet is rotating at a constant angular speed ω counterclockwise. Using a stopwatch, the rider finds it takes 3.4 seconds to go from the lowest point on the ride to a point Q, which is level with the top of a 44 ft pole. Assume the lowest point of the ride is 3 feet above ground level.



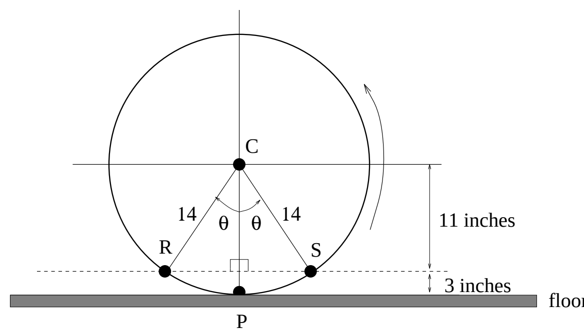
(a) What is the angular speed ω? How fast is the rider moving in mph?

(b) Find parametric equations for the motion of a rider on the wheel, assuming the rider begins at the lowest point on the wheel.

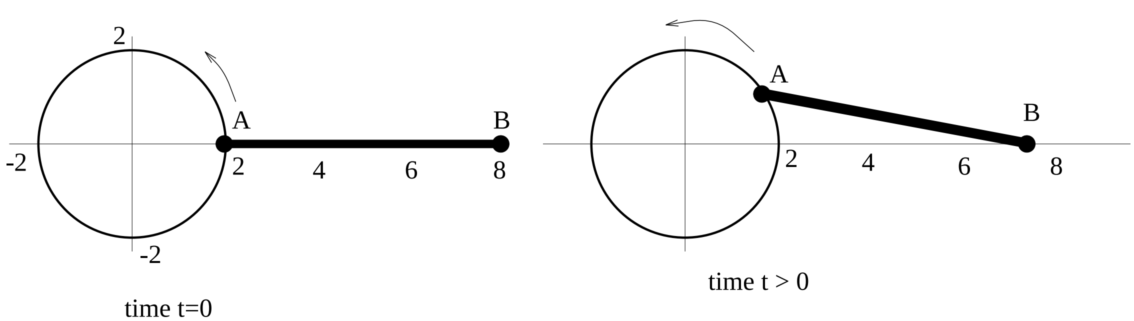
(c) Due to a malfunction, the ride abruptly stops 35 seconds after it began. To the nearest foot, how high above the ground is the rider?

7. In Example 3.2.3, we studied riding a stationary exercise bike at a steady speed of 40 mph. The rear wheel is 28 inches in diameter. If a pebble sticks in the tread of the rear wheel, describe parametric equations for the motion of the pebble around the center of the wheel.

8. Maintain the situation in the previous problem. Again, a pebble sticks to the rear tire and the rider pedals to maintain a constant 40 mph speed. During a typical revolution of the rear wheel, describe where the pebble is located when it is 3 inches above the level of the floor. During one revolution of the rear wheel, how much time does the pebble spend at least 3 inches above the floor level? Here is a picture:



9.* A six foot long rod is attached at one end A to a point on a wheel of radius 2 feet, centered at the origin. The other end B is free to move back and forth along the x-axis. The point A is at $(2, 0)$ at time $t = 0$, and the wheel rotates counterclockwise at constant speed with a period of 3 seconds.



(a) As the point A makes one complete revolution, indicate in the picture the direction and range of motion of the point B.

(b) Find the coordinates of the point A as a function of time t.

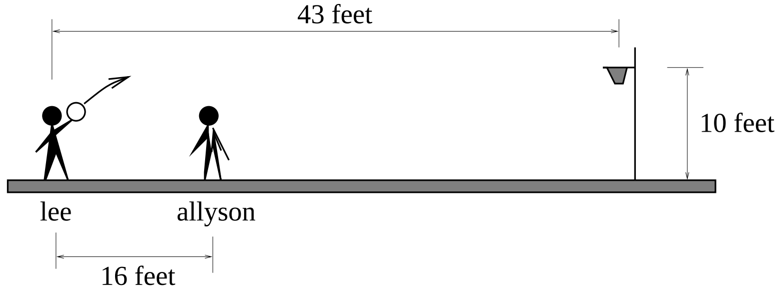
(c) Find the coordinates of the point B as a function of time t.

(d) What is the x-coordinate of the point B when $t = 1$?

(e) Find the first two times when the x coordinate of the point B is 5.

(f) Use a graphing device to sketch the graph of the function in c. on the domain $0 \leq t \leq 12$. Is this a sinusoidal function?

10. Lee has been boasting all year about his basketball skill and Allyson can't stand it anymore. So, she has challenged Lee to a game of one-on-one. With 2 seconds to go, Allyson is leading 14-12 and Lee fires a desperation three. Impose coordinates with Lee's feet at the origin and use units of feet on each axis. The path of the ball is described by the parametric equations: $x(t) = 28.925t, y(t) = -16t^2 + 34.472t + 6$.



(a) How high is the ball when Lee fires his shot?

(b) When is the ball directly over Allyson?

(c) Allyson can jump and block a ball 9.5 feet above the floor. Can Allyson block Lee's shot?

(d) Where and when is the ball 22 feet above the floor?

(e) Where and when does the ball reach it's highest point above the floor?

(f) Who wins the game?

(g) Solve the equation $x = x(t)$ for t in terms of x, then plug this into the equation for $y = y(t)$. Sketch the graph of the resulting function $y = f(x)$; this is the path of Lee's shot.

11. The population of caribou and wolves in a remote Alaskan valley is modeled by

$$x(t) = 100\sin(\frac{2\pi}{7}(t-6)) + 200 \quad \text{(caribou)}$$
$$y(t) = 100\sin(\frac{2\pi}{7}(t-\frac{1}{2})) + 200 \quad \text{(wolves)},$$

where t represents years since 1970.

(a) Sketch the graphs of both population models on the same axes, for times between 1970 and 2000. Clearly label the axes on your graphs.

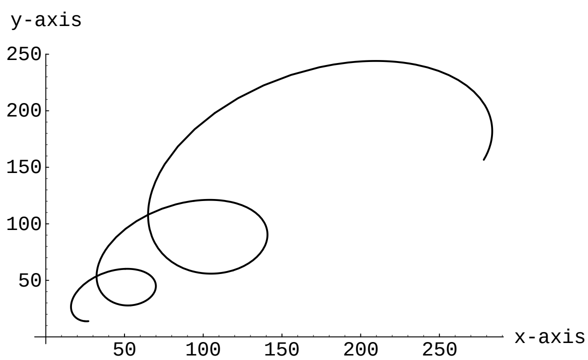
(b) The wolves can be regarded as predators and the caribou as prey. In a., the predator graph "lags" the prey graph. Describe what this means in words.

(c) Using a., how often will the predator population exceed the prey population?

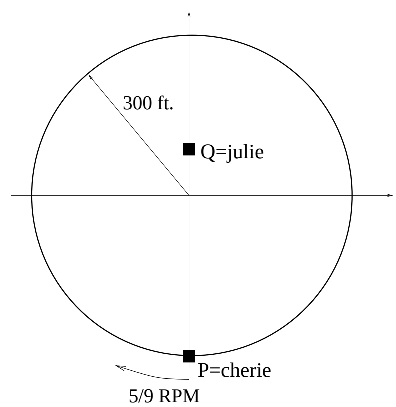
(d) Use a graphing device to sketch the parametric equations $A(t) = (x(t), y(t))$ in the xy-coordinate system. Do this for three different time domains: $0 \leq t \leq 3$, $0 \leq t \leq 5$, $0 \leq t \leq 7$, $0 \leq t \leq 26$.

(e) Using d., plot the point $A(0) = (x(0), y(0))$. Describe the motion of $A(t)$ around the plot for $0 \le t \le 7$ and interpret what is happening to the predator/prey population.

(f)* The situation described in a.-e. is "cyclic"; i.e. the population of each species cycles sinusoidally. Suppose that the population models were *different* functions $x^*(t)$ and $y^*(t)$ and that the graph of $A^*(t) = (x^*(t), y^*(t))$ is as pictured below. Explain what is happening. (Note: $A(0)$ is the right-most endpoint of the curve.)



12. Cherie is running clockwise around a circular track of radius 300 feet. She starts at the location pictured running with an angular speed of $\frac{5}{9}$ RPM clockwise. Julie stands at the pictured location, 100 feet from the center of the circle.



(a) When will Cherie first reach the location on the track closest to Julie?

(b) Where is Cherie located in 40 seconds? (i.e. find her coordinates). Indicate this point in your picture and label it as $P(40)$.

(c) How far has Cherie traveled (distance she has run) in 40 seconds?

(d) Let $d(t)$ be the function that calculates the distance between Cherie and Julie at time t seconds.

i. If the domain is taken to be the time required for Cherie to complete one revolution, what is the domain and range of $d(t)$?

ii. Write down a formula to calculate $d(t)$.

(e)* Assume that Julie is instead located at the position $(0, a)$, where $0 \le a \le 300$ is a given constant. (So part (d) was the case when $a = 100$.)

i. Find a function $d_a(t)$ that computes the distance between Julie and Cherie at time t. Your formula will involve t and a and should collapse to the formula for $d(t)$ in (d) if you set $a = 100$.

ii. Use a graphing device to sketch the graphs of $d_a(t)$ on the domain $0 \le t \le 216$ seconds, for $a = 0, 50, 100, 200, 300$. Are these sinusoidal functions?

13. Suppose that $x(t)$ and $y(t)$ are both linear functions in t. What can you say about the nature of the parametrized curve $P(t) = (x(t), y(t))$?