3.8 The Addition Formulas

1. Find the exact values of $\cos(\frac{5\pi}{12})$ and $\sin(\frac{11\pi}{12})$.

2. Write the expression $\cos(48^o)\cos(23^o) + \sin(48^o)\sin(23^o)$ as a function of one angle.

3. Use an addition formula to find the solutions of the equation $\sin(4t)\cos(t) = \sin(t)\cos(4t)$. on the domain $[0, 2\pi)$.

4. Express $f(x) = \cos(4x) - \sqrt{3}\sin(4x)$ in the form $A\cos(Bx - C)$ and standard sinusoidal form, then sketch the graph.

5. If a mass suspended from a spring is raised y_0 feet and given an initial vertical velocity of v_0 feet per second, the position y of the mass t seconds later is given by

$$y = y_0\cos(\omega t) + \frac{v_0}{\omega}\sin(\omega t),$$

where ω is a positive constant related to the size of the mass and the stiffness of the spring.

   (a) If $y_0 = 2$, $v_0 = 6$, and $\omega = 3$, write y in the form $A\cos(Bt - C)$. What is the amplitude of y?

   (b) If $y_0 = 1$ and $\omega = 2$, what initial velocity results in an amplitude of 4 feet?

6. If a tuning fork with frequency ω is struck and held at a certain distance from the eardrum, the resulting pressure of the sound wave on the eardrum at time t is given by $p_1(t) = c\sin(\omega t)$ for some constant $c > 0$. If a second identical tuning fork is struck with a possibly different force and held at a possibly different distance from the eardrum, the resulting pressure on the eardrum is given by $p_2(t) = d\sin(\omega t + l)$ for some constants d and l with $d > 0$ and $-\pi \le l \le \pi$. When both forks are struck, the total pressure is the sum of these two:

$$p(t) = p_1(t) + p_2(t) = c\sin(\omega t) + d\sin(\omega t + l).$$

   (a) Express $p(t)$ in the form $a\cos(\omega t) + b\sin(\omega t)$ for some suitable constants a and b.

   (b) Show that the amplitude A of $p(t)$ is given by

$$A^2 = c^2 + d^2 + 2cd\cos(l).$$

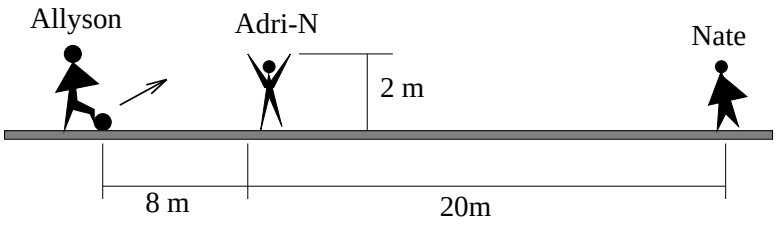
   (c) If the forks are struck with equal force, so that $c = d$, show that $A = 2c|\cos(\frac{l}{2})|$. For which values of l do we have $A > c$ (constructive interference)? For which values do we have $A < c$ (destructive interference)?

7. If a projectile is fired from ground level with an initial speed v ft/sec at an angle θ with the horizontal, the distance R feet it travels before hitting the ground is given by

$$R = \frac{v^2}{16}\sin(\theta)\cos(\theta).$$

If $v = 80$ ft/sec, find the angle θ that makes $R = 200$ feet. (Hint: use a double angle formula.)

8. Allyson needs to kick a soccer ball over Adri-N's head right to Nate; the three players are at the indicated locations. Determine the initial speed and angle Allyson should kick the ball. (Note: There is more than one correct answer; make sure to indicate what led you to your answer.)



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